

# **IMPROVING THE COST MODEL DEVELOPMENT PROCESS USING FUZZY LOGIC**

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## **ABSTRACT**

Global competition has correspondingly increased the required number of new cost models. These new cost models must be rapidly built, use less data, operate using expert opinion, and be understandable to a wide variety of product team members.

The cost model development process consists of data identification, data collection, and data analysis tasks. Fuzzy logic is considered as a new method to fulfil the new requirements in cost model development. Fuzzy logic can be built using data or words, and can be similarly understood by users.

The fuzzy logic methods of Mamdani, a subtractive clustering based algorithm, and the Adaptive Neuro-Fuzzy Inference System are chosen from a review of the fuzzy logic literature, to be compared with Multiple Linear Regression analysis. These methods are subsequently tested on a non-linear and linear function form, comparable to cost model forms found in the literature. Different configurations of the fuzzy logic methods are tested and built efficiently through the Taguchi methodology. Dependant on the error measures of Average Percentage Error and Average Absolute Error, each method is placed in a favourable light through a particular chosen configuration by the Taguchi methodology. It is found that all methods were capable in at least one instance of estimating a cost model function to a definitive level, a level suitable for commercial quotations. In addition there was also a possible range of errors dependant on the fuzzy logic model configuration.

The numerical results are used to assist in the development of a proposed advisory process for cost engineers using fuzzy logic. The process leverages case studies from the literature, the numerical results from the experiments, and knowledge obtained through the research that is captured within the thesis.



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I would finally like to thank my family and friends for their amazing support.

## **Declaration**

I declare that the work described within this thesis was originally undertaken by myself, (Paul Baguley), between the dates of registration for the degree of Doctor of Philosophy at De Montfort University, February 1999 to December 2004.

# CONTENTS

<b>ABSTRACT.....</b>	<b>I</b>
<b>ACKNOWLEDGEMENTS.....</b>	<b>II</b>
<b>DECLARATION.....</b>	<b>III</b>
<b>TABLE OF CONTENTS.....</b>	<b>IV</b>
<b>LIST OF FIGURES.....</b>	<b>IX</b>
<b>LIST OF TABLES.....</b>	<b>XI</b>
<b>GLOSSARY.....</b>	<b>XIII</b>
<b>CHAPTER 1 INTRODUCTION.....</b>	<b>1</b>
1.1 Introduction.....	2
1.2 What is Cost Modelling?.....	4
1.3 How Cost Modelling Affects Businesses and Cost Accountancy.....	5
1.4 Cost Estimating Versus Cost Modelling.....	8
1.5 Fuzzy Logic.....	10
1.6 Aims of the Research.....	11
1.7 Objectives of the Research.....	11
1.8 The Origins of the Research Aims.....	12
1.9 Chapters Overview.....	13
<b>CHAPTER 2 THE COST MODEL DEVELOPMENT PROCESS...</b>	<b>16</b>
2.1 Introduction.....	17
2.2 Cost Model Development Tasks.....	17
2.3 Cost Model Characteristics.....	19
2.4 Data Identification.....	20
2.5 Data Collection.....	24
2.6 Data Analysis.....	33
2.7 Subjective Judgement.....	46
2.8 Assumptions.....	54
<b>CHAPTER 3 FUZZY LOGIC.....</b>	<b>57</b>
3.1 Introduction.....	58
3.2 Fuzzy Logic Basics.....	58
3.3 Structure of Fuzzy Logic Models.....	60
3.3.1 Fuzzification.....	61
3.3.2 Rules.....	62
3.3.3 Inference.....	64
3.3.4 Defuzzification.....	65
3.4 Comparison of Alternative Fuzzy Logic Methods: to Show the Variety of Forms of Fuzzy Logic.....	65
3.4.1 Mamdani, TSK, Adaptive Neuro-Fuzzy Inference System and Subtractive Clustering Methods.....	66
3.4.2 Mamdani Method.....	67
3.4.3 Takagi Sugeno Kang Method.....	68
3.4.4 Adaptive Neuro-Fuzzy Inference System (ANFIS).....	69
3.4.5 Subtractive Clustering.....	69
3.5 Significance of the Shape of a Fuzzy Set.....	70
3.6 Fuzzy Inference System Structural Elements Used in Previous Research.....	71
3.6.1 Fuzzification.....	72



3.6.2	Type of fuzzy Sets.....	72
3.6.3	Number of Fuzzy Sets.....	75
3.6.4	Parameters of Fuzzy Sets.....	76
3.6.5	Overlap, Shoulder Width and Symmetry of Fuzzy Sets.....	78
3.6.6	Inference.....	78
3.6.7	Implication.....	78
3.6.8	Rules.....	79
3.6.9	Defuzzification.....	82
3.7	Influence of Fuzzy Logic on Cost Model Characteristics.....	84
3.7.1	Estimating Accuracy.....	84
3.7.2	User Personnel.....	85
3.7.3	Set-up and Operating Costs.....	87
3.7.4	Data Requirements.....	89
3.8	The Cost Model Characteristics in Terms of Previous Research into Using Fuzzy Logic.....	90
3.8.1	Accuracy.....	91
3.8.2	Data Collection Effort.....	92
3.8.3	Subjective Judgement.....	93
3.8.4	Development.....	94
3.8.5	Tasks.....	94
3.8.6	Detail of Input Data.....	95
3.8.7	Estimate Application Time.....	96
3.8.8	Personnel Whom Operate the System.....	97
3.8.9	Cost Model Level.....	97
<b>CHAPTER FOUR EXPERIMENTAL DESIGN.....</b>		<b>99</b>
4.1	Introduction.....	99
4.2	Design of Experiments.....	104
4.3	Building the Orthogonal Array.....	107
4.4	Reference Cost Model.....	107
4.5	Method of Building the Fuzzy Logic Models.....	109
4.6	Description of the Experimental Trials.....	110
4.6.1	Experimental Trials 1: Triangular and Trapezoidal Membership Functions: Shape Based Defuzzification Methods (“The Basic Experiment”).....	110
4.6.2	Experimental Trials 2: Triangular and Trapezoidal Membership Functions: Maximum Value Based Defuzzification Methods (“Alternative Defuzzification Methods”).....	111
4.6.3	Experimental Trials 3: Gaussian Membership Functions: Shape Based Defuzzification Methods (“Using Gaussian Membership Functions”).....	112
4.6.4	Experimental Trials 4: Gaussian Membership Functions: Maximum Value Based Defuzzification Methods (“Gaussian Membership Functions” and “Changing the Defuzzification Methods”).....	113
4.6.5	Experimental Trials 5: “Adding Membership Functions in the Output”.....	113
4.6.6	Experimental Trials 6: “Using a Neural Network” 1.....	115



4.6.7 Experimental Trials 7: “Using a Neural Network” 2..... 116

4.6.8 Experimental Trials 8: “The Clustering Method:  
Subtractive Clustering”..... 118

4.6.9 Experimental Trials 9: Multiple Linear Regression Analysis..... 120

4.6.10 Experimental Trials 10: “Testing a Linear Cost Model”..... 121

4.6.11 Experimental Trials 11: “Testing for Interactions”..... 121

**CHAPTER FIVE EXPERIMENTAL TRIALS RESULTS.....123**

5.1 Introduction..... 124

5.2 Experimental Trials 1: Triangular and Trapezoidal Membership  
Functions: Shape Based Defuzzification Methods..... 125

5.3 Experimental Trials 2. Triangular and Trapezoidal Membership  
Functions: Maximum Value Based..... 126

5.4 Experimental Trials 3: Gaussian Membership Functions:  
Shape Based Defuzzification Methods..... 127

5.5 Experimental Trials 4: Gaussian Membership Functions:  
Maximum Value Based Defuzzification..... 128

5.6 Experimental Trials 5: Increasing the Number of Consequent  
Membership Functions and Hence Decreasing the Imprecision  
of Each One..... 129

5.7 Experimental Trials 6: Using a Neural Network 1, Including  
Comparing Using Multiple Linear Regression (MLR)  
(from Experimental Trials 9) with Varying Data Points to Using an  
Adaptive-Neuro Fuzzy Inference System (ANFIS)..... 130

5.8 Use of the Product Rule in Selected Models from Experimental  
Trials 1-4..... 132

5.9 Experimental Trials 7: Using a Neural Network 2, Adaptive  
Neuro-Fuzzy-Inference-System..... 132

5.10 Experimental Trials 8: Subtractive Clustering, Including Comparing  
using Multiple Linear Regression (MLR) (from Experimental  
Trials 9)..... 134

5.11 Experimental Trials 10: Testing the Effect of the Best Model  
Structure from Experimental Trials 4 on a Linear Model..... 139

5.12 Experimental Trials 11: Identifying Interactions in Experimental  
Trials 9: Subtractive Clustering..... 139

5.13 A Change in Error Measure..... 142

**CHAPTER SIX DISCUSSION..... 143**

6.1 The Need for a New Cost Model Development Data  
Analysis Method..... 144

6.1.1 Environment: the Need for a New Cost Modelling Method..... 144

6.2 Cost Model Development Methods..... 145

6.2.1 Cost Model Development: Why use Fuzzy Logic? ..... 145

6.3 Fuzzy Logic structure..... 146

6.3.1 Fuzzy Logic Structure..... 146

6.3.2 Systematic methods..... 149

6.3.3 Choice of Fuzzy Logic Methods..... 150

6.3.4 Experimental Results: Estimating Accuracy as Related  
to Structural Elements..... 152

6.3.5 Experimental Results: Initial Observations..... 152



6.3.6	Systematic Method to Choose Fuzzy Logic Methods and Structural Elements: Best Model and Best Structure Problem.....	153
6.3.7	Trends Obtained from the Taguchi Methodology as a Systematic Method.....	154
6.4	Experimental Trials: Key Results.....	155
6.4.1	Deriving Meaning from the Models and Related Problems.....	162
6.4.2	Systematic Methods: a Closer Look at the Structural Elements used in the Numerical Experiments.....	164
6.4.3	Regression Versus Fuzzy Logic Models.....	165
6.4.4	Non-Linear and Linear Models.....	166
6.4.5	Deriving Meaning of the Subtractive Clustering Models.....	167
6.4.6	Observations of the Numerical Results.....	168
6.4.7	Standard Deviation.....	169
6.4.8	Error Measures.....	169
6.5	The Effect of Fuzzy Logic on Cost Model Development.....	171
6.5.1	Range of Application Areas.....	171
6.5.2	Validation: Existing Applications of Fuzzy Logic Related to Costing have all been Relatively Successful.....	171
6.5.3	Cost Model Development Process, Data Identification, Data Collection, and Data Analysis: Advantages and Disadvantages of using Fuzzy Logic.....	174
6.5.4	A Proposed Decision Making Methodology for Choosing Fuzzy Logic.....	177
6.5.5	Experiment Based.....	178
6.5.6	Rule Base Formation.....	178
6.5.7	The Proposed Decision Making Methodology for Applying Fuzzy Logic by Cost Engineers.....	179
6.6	Advantages and Disadvantages of using Fuzzy Logic for Cost Engineering: Summary.....	181
6.6.1	Advantages.....	181
6.6.2	Disadvantages.....	181
6.7	Sequence of Steps in using the Proposed Decision Making Methodology Advisor.....	181
<b>CHAPTER SEVEN CONCLUSION.....</b>		<b>183</b>
<b>CHAPTER EIGHT FURTHER WORK.....</b>		<b>188</b>
<b>REFERENCES.....</b>		<b>191</b>
<b>BIBLIOGRAPHY.....</b>		<b>211</b>
<b>APPENDIX A1 FUZZY LOGIC THEORY.....</b>		<b>216</b>
A1.1	Formal Definition of a Fuzzy Set.....	216
A1.2	Structure of the Four Methods of Mamdani, Takagi Sugeno Kang, Adaptive Neuro-Fuzzy Inference System, and Subtractive Clustering.....	216
A1.3	Formal Definition of Defuzzification Methods.....	217
A1.4	Formal Definition of Some Fuzzy Logic Operators.....	218
A1.5	Mathematical Description of Some Popular Membership Functions...	220
A1.6	Alpha Decomposition.....	220
A1.7	Structure of Adaptive Neuro-Fuzzy Inference System.....	221
A1.8	Sample of Universal Function Approximation Proofs.....	222



A1.9 The Formal Definition of Zadeh’s Extension  
Principle (Jang et al 1996)..... 222

A1.10 Example Interactions within a Fuzzy Inference System..... 223

**APPENDIX A2 SAMPLE OF FUZZY LOGIC CASE BASE  
FROM THE LITERATURE..... 225**

**APPENDIX A3 APPENDIX FOR CHAPTER 3..... 240**

A3.1 The Mathematics of Inference..... 240

**APPENDIX A4 APPENDIX FOR CHAPTER 4..... 243**

A4.1 Method of Building the Mamdani Method Models..... 243

A4.2 Experimental Tables..... 250

**APPENDIX A5 APPENDIX FOR CHAPTER 5..... 257**

A5.1 Numerical Results Tables..... 257

**APPENDIX A6 APPENDIX FOR CHAPTER 6..... 277**

A6.1 Trends Obtained from the Taguchi Methodology as a Systematic Method.. 277

A6.2 Problems with Systematic Method: What Interactions?..... 279

A6.3 Observations of the Numerical Results..... 281

A6.4 Experimental Methods: the Other Cost Model Characteristics..... 282

A6.5 Cost Model Characteristics for Fuzzy Logic..... 283

A6.6 Personnel Involved..... 283

A6.7 Data Requirements, Model Development and Operating Costs..... 284

A6.8 Cost Model Development Process, Data Identification, Data Collection  
and Data Analysis: Advantages and Disadvantages of Using Fuzzy Logic. 285

A6.9 Experiment Based..... 287

A6.10 Observation of Results..... 289

A6.10.1 Average Percentage Error (APE)..... 289

A6.10.2 Average Absolute Error (AAE)..... 289

A6.11 Regression Versus Fuzzy Logic Models.....292

**PUBLISHED PAPERS.....299**



## LIST OF FIGURES

Figure 2.1: Structure of the Research.....	17
Figure 2.2: Cost Model Development Process (Wang 2000).....	18
Figure 3.1: Fuzzy Sets from a Screenshot in MATLAB (the Labels Commonly Stand for Low, Nearly Low, Medium, Nearly High, and High).....	59
Figure 3.2: Examples of Fuzzy Set Shapes in MATLAB.....	60
Figure 3.3: Structure of a Fuzzy Inference System.....	61
Figure 3.4: Rule Reduction Through Hierarchies.....	64
Figure 3.5: Liao's (1996) Fuzzy Sets.....	73
Figure 3.6: Example of a Type-2 Fuzzy Set in which all Degrees of Membership are Themselves a Fuzzy Set.....	89
Figure 4.1: Use of Factors and Levels to Implement Fuzzy Logic Methods.....	100
Figure 4.2: An Overview of all Fuzzy Logic Experiments.....	102
Figure 4.3: Overview of the Mamdani Method Experiments.....	103
Figure 4.4: Linear Cost Model.....	104
Figure 4.5: Trapezoid and its Shoulder Width.....	111
Figure 4.6: Imprecision.....	114
Figure 4.7: Increasing the Imprecision.....	115
Figure 4.8: Summary of GENFIS then ANFIS.....	116
Figure 4.9: GENFIS1 Builds an Initial Fuzzy Inference System.....	117
Figure 4.10: A Cluster Forms a Rule.....	120
Figure 4.11: An Example Linear Graph.....	122
Figure 5.1: Gaussian Membership Functions with Decreasing Imprecision.....	129
Figure 5.2: Gaussian Membership Functions with Adaptive Neuro Fuzzy Inference System (ANFIS) Defuzzification Compared to Regression Analysis via Average Percentage Error.....	131
Figure 5.3: Gaussian Membership Functions with ANFIS Defuzzification Compared to Regression Analysis via Average Percentage Standard Deviation (APSD).....	131
Figure 5.4: Best Model and Best Structure of the ANFIS in Experimental Trials 7.....	133
Figure 5.5: Number of Rules Versus MAPE from Clustering Run 2.....	135
Figure 5.6: Number of Rules Versus MAPE Without Outliers.....	135
Figure 5.7: Comparison of the MAPE of the Best Model and Best Structure of the Subtractive Clustering Method and Multiple Linear Regression for Varying Number of Data Points.....	137
Figure 5.8: Standard Deviation of MLR and Subtractive Clustering Best Model	138
Figure 5.9: Standard Deviation of MLR and Subtractive Clustering Best Model Over a Reduced Range.....	138
Figure 5.10: Graphs to Show Interaction Effects of Cluster Influence For Each Input Variable.....	140
Figure 6.1: Measures of Imprecision.....	158
Figure 6.2: Cost Model 7, Experimental Trials 1.....	159
Figure 6.3: Example Output Fuzzy Sets: Cost Model 9 Experimental Trials 4..	167
Figure 6.4: Maximum Based Defuzzification Methods are	



Capable of Unpredictable Results..... 176

Figure 6.5: The Proposed Decision Making Methodology for  
Cost Engineering With Fuzzy Logic..... 180

Figure A1.1: Adaptive Neuro Fuzzy Inference System Structure..... 220

Figure A1.2: Interactions Between Fuzzy Set Shape and Defuzzification,  
Example One..... 223

Figure A1.3: Interactions Between Fuzzy Set Shape and Defuzzification,  
Example Two..... 224

Figure A4.1: Fuzzification of a Data Point..... 244

Figure A4.2: How Overlap is Determined in the Experimental Trials, e.g.  
Experimental Trials 1..... 246

Figure A4.3: Data Points Become Fuzzy Sets..... 246

Figure A4.4: Values that have a Certainty of One are Used to  
Generate Values of Y..... 248

Figure A4.5: Choosing the Fuzzy Sets for Y in the Output in Developing Rules... 249

Figure A4.6: Systematically Building a Rule Base..... 249

Figure A4.7: Constructing the Rule Base..... 249

Figure A5.1: Experimental Trials 1: Relative Effect of Structural Elements..... 260

Figure A5.2: Comparison of Defuzzification Methods from Experimental  
Trials 1 (Centroid, Bisector), and 2 (LOM, SOM,  
MOM)..... 263

Figure A5.3: Experimental Trials 3: Effect of Fuzzy Logic Structural Elements... 265

Figure A5.4: Experimental Trials 3 and 4 Defuzzification Methods..... 267

Figure A6.1: Explaining the Presence of Interactions..... 280

Figure A6.2: Overlap is Spread over the Range..... 281

Figure A6.3: Output Variables in MATLAB..... 294

Figure A6.4: Input Variables from a Subtractive Clustering Model..... 295

Figure A6.5: Input Fuzzy Sets for X1 in “Best Model” for “Experimental Trials 9:  
Subtractive Clustering”..... 296

Figure A6.6: Output Linguistic Terms (Linear Functions Optimised by  
Linear Least Squares) from the Subtractive Clustering Based Method..... 298



## LIST OF TABLES

Table 1.1: Changes Occurring in the Manufacturing Sector (Wang 2000).....	3
Table 1.2: Changes in the Cost Model Development Process.....	4
Table 2.1: Cost Model Characteristics and Issues.....	19
Table 2.2: Machining.....	43
Table 2.3: Injection Moulding Time.....	44
Table 2.4: Powder Metal Processing.....	44
Table 3.1: Methods for Creating Fuzzy Sets (Ross 1995).....	62
Table 3.2: Types of Fuzzy Logic.....	66
Table 3.3: A Direct Comparison Between the Mamdani, TSK, Subtractive Clustering, and ANFIS Methods.....	67
Table 4.1: The Fuzzy Logic Methods and Their Associated Factors and Levels Used in a Design of Experiments.....	101
Table 4.2: Fuzzy Logic Structural Elements for the Mamdani Method.....	105
Table 4.3: Example of Cost Data Used to Develop Fuzzy Logic Models.....	109
Table 6.1: Categorising of Fuzzy Logic Methods.....	159
Table 6.2: A Summary of the Numerical Experimental Results Through Ranking.....	161
Table 6.3: The Meaning of Fuzzy Sets in Experimental Trials 1-4.....	163
Table 6.4: A Summary of Observations Linking the Fuzzy Sets and their Structural Elements to Cost Engineering Issues.....	165
Table A1.1: Mamdani Method Structure.....	116
Table A1.2: Takagi Sugeno Kang Method Structure.....	117
Table A1.3: Adaptive Neuro Fuzzy Inference System Method Structure.....	117
Table A1.4: Subtractive Clustering Based Method Structure.....	117
Table A1.5: Universal Function Approximation Proofs.....	121
Table A4.1: Design For Experimental Trials 1 (“The Basic Experiment”).....	251
Table A4.2: Alternative Defuzzification Methods.....	252
Table A4.3: Gaussian Membership Functions.....	252
Table A4.4: “Gaussian Membership Functions” and “Changing the Defuzzification Methods”.....	253
Table A4.5: Taguchi Orthogonal Array for Experimental Trials 7.....	253
Table A4.6: An L16 for Subtractive Clustering.....	254
Table A4.7: Taguchi L16 Orthogonal Array used with the Best Structure from the Mamdani Method.....	255
Table A4.8: The L16 Table Used to Identify Interactions. Column Numbers are Shown at the Top of Each Column.....	256
Table A5.1: Experimental Trials 1: Results of Individual Trials.....	257
Table A5.2: Experimental Trials 1: Effect of Individual Structural Elements.....	258
Table A5.3: The Numerical Effect of Individual Structural Elements.....	258
Table A5.4: Experimental Trials 1: The Effect of Changing Random Number Sets	259
Table A5.5: Experimental Trials 2: Individual Trials.....	262
Table A5.6: Experimental Trials 2: Effect of Individual Structural Elements.....	262
Table A5.7: Experimental Trials 3: Individual Trials.....	263
Table A5.8: Experimental Trials 3: Effect of Individual Structural Elements.....	264
Table A5.9: Experimental Trials 4: Results of Individual Trials.....	266
Table A5.10: Experimental Trials 4: Effect of Individual Structural Elements.....	266



Table A5.11: Use of the Product Rule in Experimental Trials 1-4.....	267
Table A5.12: Adaptive Neuro-Fuzzy Inference System.....	268
Table A5.13: Experimental Trials 7: Effect of Individual Structural Elements.....	268
Table A5.14: The Effect of Varying the Number of Data Points for the Best Model / Best Structure.....	269
Table A5.15: Results of Individual Clustering Trials.....	270
Table A5.16: Effect of Individual Structural Elements.....	271
Table A5.17: Comparison of Random Number Sets.....	271
Table A5.18: Individual Trials Testing the Best Model from Table 5.9 in Estimating Linear Models Chosen Using the Orthogonal Array in Table 5.19.....	272
Table A5.19: Taguchi Orthogonal Array to Choose the Linear Models for Which to Test the Best Model from Table 5.9.....	273
Table A5.20: The use of MLR Method to Estimate the Linear Model Equation 4.3, and Varying the Number of Data Points.....	274
Table A5.21: Experimental Trials 11, Investigating Interaction Effects for Subtractive Clustering.....	274
Table A5.22: The Average MAPE for Cluster Influence for Variables X1, X2, X3 and X4 and Different Levels of Interactions (X1*X2), (X1*X3), (X1*X4), (X2*X3), (X2*X4), (X3*X4).....	275
Table A5.23: Repeat Experiments with Absolute Instead of Percentage Errors...	276
Table A6.1: Cost model Characteristics in the Numerical Experiments and What They are Affected by (Chapter 4).....	283
Table A6.2: Categorising of Fuzzy Logic Methods.....	287
Table A6.3: Categories within the Decision Making Methodology for Cost Engineering with Fuzzy Logic.....	288
Table A6.4: Number of Rules per Cost Model in Experimental Trials 9, Subtractive Clustering.....	293

## GLOSSARY

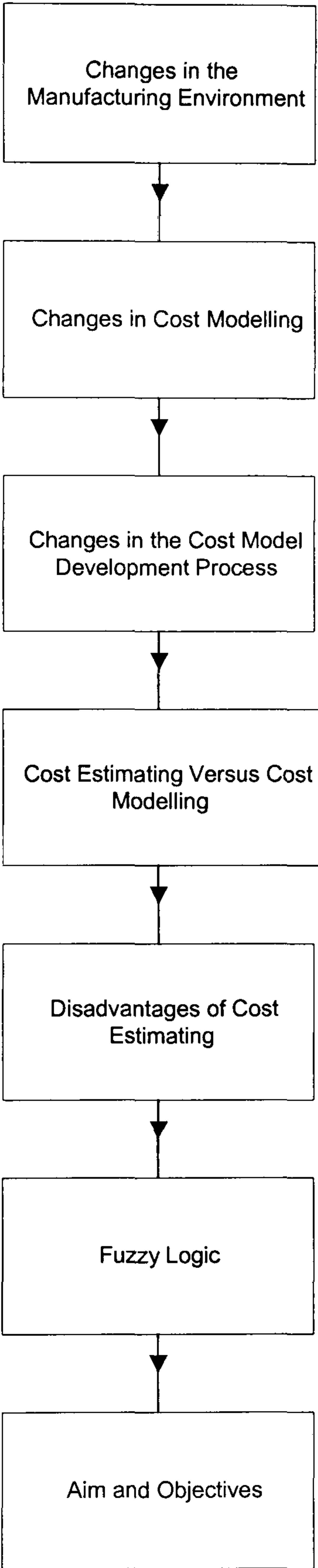
Average Absolute Error	(AAE)
Average Absolute Standard Deviation	(AASD)
Activity Based Costing	(ABC)
Automated Guided Vehicle	(AGV)
Adaptive Neuro-Fuzzy Inference System	(ANFIS)
Artificial Neural Network	(ANN)
Average Percentage Error	(APE)
Average Percentage Standard Deviation	(APSD)
Automatic Speech Recognition	(ASR)
Concentration Contrast Relaxation	(CCR)
Cost Estimating Relationship	(CER)
Cambridge Material Selection	(CMS)
Computer Numerical Control	(CNC)
CONstructive COst MOdel	(COCOMO)
Cost Optimisation Software for Transport	
Aircraft Design Evaluation	(COSTADE)
Data Analysis	(DA)
Data Collection	(DC)
Dilution Contrast Intensification	(DCI)
Design For Manufacture and Assembly	(DFMA)
Data Identification	(DI)
Fuzzy Associative Memory	(FAM)
Fuzzy C Means	(FCM)
Fuzzy Inference System	(FIS)
Flat Plate Processing	(FPP)
Integrated computer-aided manufacturing DEFINition	(IDEF)
Genetic Algorithm	(GA)
General Error Regression Model	(GERM)
Graphical User Interface	(GUI)
Life Cycle Cost	(LCC)
Largest Of Maximum	(LOM)
Mean Average Percentage Error	(MAPE)
Model Identification Process	(MIP)
Multiple Linear Regression	(MLR)
Mean Of Maximum	(MOM)
Maynard Operations Sequence Technique	(MOST)
Mean Time Between Failures	(MTBF)
Methods Time Measurement	(MTM)
National Aeronautics and Space Administration	(NASA)
Probability Density Function	(pdf)
Predetermined Motion Time Study	(PMTS)
Rough Order of Magnitude	(ROM)
Robot Time and Motion	(RTM)
Society of Cost Estimating and Analysis	(SCEA)
Subjective Judgement	(SJ)
Smallest Of Maximum	(SOM)





# CHAPTER ONE

## INTRODUCTION



## 1.1 Introduction

The manufacturing sector is currently undergoing rapid change brought about by such factors as globalisation, increased levels of competition, technological breakthroughs, legislation for the environment and a greater awareness of cost and waste. These changes within the manufacturing sector lead to:

- **Changes in strategy.** For example Hoon and Sim (1996) identified the need for time-based competitive strategies in response to a need for shorter development times and greater variety within products.
- **Increased difficulty in controlling businesses.** A need for increased levels of control can be inferred from Bode (2000) when he identifies the control requirements for “reducing product lifecycles, differentiating markets, increasing product complexity, rapidly changing technological knowledge, and reacting to higher customer sophistication”. This difficulty is also identified in the military aerospace sector by Rush and Roy (2001b) who state that: “market pressures are increasing, and rapid advances in technology increase the requirement for more complex, multi-role products, which lead to spiralling development costs and increased financial risks”. There is also a need for low costs with high quality as identified by Shehab and Abdalla (Shehab and Abdalla 2001, Shehab and Abdalla 2002).
- **A need to control costs to offset decreasing profit margins.** Normally controlling costs is achieved through lowering capital investment, performing less research and generating less development and product costs. However, improvements in the control of costs can often arise from reducing product development times, (Andriess 1994).



- **Delays in decision making.** Delays occur while awaiting cost estimates (Farineau et al 2001), leading to delays in product and / or process development times.

In terms of cost modelling these changes are resulting in:

- **The need for more precise and accurate cost estimates** to improve the product development process and its outputs, (Bashir and Thomson 2001).
- **Increased difficulty in cost estimating** which arises from the time reduction pressures affecting development processes. Here Scanlan et al (2002) identified that available development time is used to determine the most suitable approach to costing, i.e. a low level of detail (*level one*) was used when available time was short and a high level (*level three*) when sufficient development time was available. Here *level one* indicates the use of say, parameterised overall dimensions, product specifications and primary materials; *level two* indicates the use of geometry of major parts and assembly relationships; and *level three* the use of detailed geometry and tolerances. In terms of costing methods *level one* employed “parametric costing” to establish cost estimating relationships, level three used “bottom up costing” and level two made use of a combination of both “parametric” and “bottom up” costing. Wang (2000) is used to help sum the changes within the manufacturing environment, i.e. Table 1.1.

**Table 1.1: Changes Occurring in the Manufacturing Sector (Wang 2000).**

- |  |
|--|
| <ul style="list-style-type: none"> <li>a) Greater choice of products demanded by customers.</li> <li>b) Greater amount of product customisation required by customers.</li> <li>c) Greater choice of materials available for use within products.</li> <li>d) Greater choice of manufacturing processes available to suppliers.</li> <li>e) Reduced product development cycles arising from increased competition within markets.</li> <li>f) Greater emphasis on minimising overall lifecycle costs of products arising from such influences as sustainable development and waste reduction.</li> </ul> |
|--|

The changes (Table 1.1) occurring in the manufacturing sector are affecting the methods by which cost models are developed. Previous research has helped to identify the main effects (Eversheim et al 1998, Wang and Stockton 2001) that have been developed as those listed in Table 1.2.

**Table 1.2: Changes in the Cost Model Development Process.**

- The need for more formalised methods of identifying the data from which models will be developed.
- The need to develop cost models earlier in the product development lifecycle.
- The availability of less historical data from which to develop models.
- The need for greater numbers of predictor variables within cost models.
- A need for greater estimating precision and accuracy to ensure cost competitive products are developed particularly when there are decreasing development cycle times.
- Recognition that single value cost estimates are not sufficient.
- The need for a cost model to be sufficiently flexible to cope when unexpected changes occur, i.e. this becomes more essential when new technologies are being cost modelled (Colmer et al 1999).
- A greater need to produce a wider range of cost model types, in terms of levels of accuracy of cost models, i.e. between level 1 and level 3 (Scanlan et al 2002).
- A cost modelling process that is more responsive to the costing needs that occur as the product changes during the product development cycle (Walter 1997).

In order to cope with the effects listed in Table 1.2, changes are required in the processes by which cost models are developed. In this respect, current cost model development processes can be described as *ad hoc*, and largely lacking coherent and consistent approaches (Roy et al 2002). In addition, traditional methods rely heavily upon the use of process and engineering experience to produce cost models.

## **1.2 What is Cost Modelling?**

Cost is a ubiquitous term in manufacturing and can be used by all including: designers, manufacturing engineers, managing directors and accountants. In this respect, Ostwald



and McLaren (2004) state: “the word cost is meaningless when used alone”, and “using precise language for the word cost aids the understanding of the constraints and conditions of the cost estimate”. Of particular importance is the making of cost related decisions in early design, as these are the ones that have the greatest impact, and potentially incur the most costs (Barton et al 2001).

Cost modelling is a process of developing a relationship, termed a Cost Estimating Relationship (CER), between cost and predictor variables, sometimes known as cost drivers (Section 2.2 defines a cost model development process in terms of generic tasks). The need for cost models arises from the need to produce cost estimates efficiently and at low cost. Cost models provide for this need by producing output cost estimates to an acceptable accuracy, from values input directly into the CER. This saves repeated and wasteful detailed analyses for each required cost estimate.

### **1.3 How Cost Modelling Affects Businesses and Cost Accountancy**

Ostwald and McLaren (2004) aid explanation of how cost estimates affect businesses. Accounting analysis examines actual business transactions after they occur, and includes concepts such as overhead, tax, budgets, and balance sheets. Operations estimating estimates the manufacturing process cost and can consist of tooling, direct labour and direct material (Gutowski (1997) describes a process time model for hand lay up of advanced composites consisting of 50 to 60 process steps whose times are related to parameters such as “area”, or “perimeter”). Product estimating uses an engineering design and includes examples such as operations costs, indirect materials and labour, and profit. Product estimating affects prices, cash flows and profits. Hence

product estimating impacts cost accounting. Cost analysis is the process of making trade-offs between design alternatives, and includes such things as cost estimates, cash flows and profit. Engineering economy is the trade-off between design alternatives when capital investments are required. This includes the time value of money and timing of cash flows.

Engineering cost analysis and estimating predicts future costs incurred whereas cost accounting deals with actual cost spent (Ostwald and McLaren 2004, Stenzel and Stenzel 2003). Cost accounting can be focussed externally, for example financial accounting, or internally, for example management accounting (Stenzel and Stenzel 2003). Management accounting methods play a role in forming cost estimates. Indeed, “basic cost accounting records” are named as a data source for developing parametric cost models in NASA’s Parametric Cost Estimating Handbook.

Example management accounting methods are (Glautier and Underdown 2001, Horngren et al 1999):

- (1) traditional cost accounting best utilised in mass production environments,
- (2) Activity Based Costing developed in order to more fully explain overhead,
- (3) lean accounting in which waste is removed from the management accounting function, and its financial and performance measures updated from previous cost accounting methods, in order to control lean production,
- (4) environmental accounting, in which the costs are evaluated of environmental impact,
- (5) future cost accounting systems for contemporary manufacturing strategy, for example virtual enterprises and agility (Gunasekaran et al 2005).



Each management accounting method can be exemplified by its cost breakdown structure and its cost elements, for example the cost pools of so called critical success factors (Gunasekaran et al 2005). The behaviour of these cost elements can be predicted from cost models developed for these elements.

Management accounting methods used by a company, form part of the assumption set of the cost model. The assumption set, recorded in the cost model documentation system, provides information on the validity of applying a cost model. Applying the cost model under broken assumptions leads to errors in cost estimates, since the cost model is no longer valid. A further situation arises when cost estimates using different management accounting methods, are utilised in building a cost model. Therefore, the assumption that cost estimates have been developed consistently, using the same management accounting methods, is broken, leading to errors. These errors are hence intrinsic to the cost model itself. This is one of the reasons that multi-organisational data sets must be used with care when forming cost models from them, since their management accounting methods can differ.

Research has provided a number of solutions to the problem of the scope and applicability of cost models. For example the development of cost models is marked by determining the “process scope” (Delgado et al 2002) or “system boundaries” (Emblemsvag 2004) of the required cost estimate from the model.

## **1.4 Cost Estimating Versus Cost Modelling**

There are several definitions of the term “cost estimating” including that by Ostwald (1974) who defines cost estimating as “concerned with cost determination and evaluation of engineering designs”. Hughes (2000) provides a more detailed definition of cost estimating as “a planned and systematic process for identifying and predicting costs within the constraints of varying levels of uncertainty and for an identified scope”.

Matthews (1983) suggests a cost estimating process that consists of

- (a) “determine what is being estimated,
- (b) break down into parts list,
- (c) determine material costs,
- (d) route individual parts,
- (e) estimate operation and set-up times,
- (f) apply labour and manufacturing-overhead rates,
- (g) calculate total manufacturing cost,
- (h) apply selling and general administrative burdens, and
- (i) apply mark-ups and develop standard selling price”.

Teng and Garimella (1998), describe summing up cost estimates of quantities, such as inventory costs, assembly costs and test, diagnostics and rework costs, in the electronics industry. Also, Boothroyd et al (2002) point to the fact that cost estimates are usually made after the detailed design stage.

A cost estimate, is therefore a specific cost value that has been associated with a specific product feature or process activity. A cost model (i.e. Cost Estimating Relationship



(CER)), however, is a general model that relates cost to the variables that affect cost (i.e. predictor variables). When specific values of predictor variables are entered into a cost model the output is a cost estimate for a specific product feature or process activity, i.e. a cost estimate. Cost models are, therefore, normally developed to enable a range of predictor variable values to be entered and hence can provide cost estimates for a range of product feature or process activity situations. Cost models can be developed using a collection of cost estimates, i.e. produce a cost estimating relationship. A number of approaches are available for establishing cost estimates, of which the main ones are:

- (a) comparative costing, e.g. using the cost estimate of a “closest case”,
- (b) subjective descriptions by experienced cost engineers, of the relationships between cost and product or process,
- (c) empirical methods that use intuition, varying amounts of detailed information and time and resource constraints,
- (d) round table estimating that uses no supporting information with all functions of the organisation represented, e.g. production engineer,
- (e) brain storming of experts that does use supporting information (Society of Cost Estimating and Analysis (SCEA)),
- (f) use of work study (e.g. time study, activity sampling, synthetic times and Pre-determined Motion Time Study (PMTS)),
- (g) use of probability distributions (Hudson 1992),
- (h) lower and upper bounds of probable cost ranges (Jha 1992), and
- (i) simulation (Zuk et al 1990)

Both Rush and Roy (2001a) and Wang (2000) have identified disadvantages with the above approaches for establishing the cost estimates, from which cost models are developed,

- (a) difficulty in understanding and / or repeating the individual tasks required to establish a cost estimate,
- (b) need to use unstructured knowledge to build a cost estimate,
- (c) it is a time consuming process,
- (d) the need for in depth experienced estimators,
- (e) accuracy of a cost estimate being dependent on the level of experience of the cost estimator rather than the method used, and
- (f) a need to frequently use the subjective opinion of estimators.

Increasing the accuracy of a cost estimate tends to increase the time and resources employed in producing it. Decreasing the accuracy occurs with an increasing use of subjective opinion that in turn decreases the time and resources for the cost estimate.

## **1.5 Fuzzy Logic**

Existing research has begun to examine a new method called fuzzy logic for use within cost estimating and modelling (for example, Jahan-Shahi et al 2001, Cox 1994, Swarc et al 1997 and Chan et al 1997). Fuzzy logic is a method that can use both expert judgement and / or data in formulating models. It is apparent, therefore, that fuzzy logic can be used in cost model development with potential for data reduction, using expert opinion and also for use at different stages of the product development life cycle. It is important to develop this new method by forming knowledge about when to use it, how



to use it and how successful it will be when it is used in different circumstances. In particular it has been identified that a decision making methodology is required to help cost estimators choose appropriate fuzzy logic methods and their structures for successful application in the cost model development process.

## **1.6 Aims of the Research**

The aims of the research are:

- to investigate fuzzy logic methods potentially suitable for use within the cost model development process,
- to assess the performance of a representative sample of fuzzy logic methods and their possible structural elements, and
- to develop a systematic treatment of applying fuzzy logic within the cost model development process

## **1.7 Objectives of the Research**

The objectives of the research are:

- to identify existing methods, and their needs, used within the cost model development process,
- to identify potential methods using fuzzy logic for use within the cost model development process to fulfil these needs,
- to identify a structure to the cost model development process, and a structure to the fuzzy logic methods,
- to identify the suitability of fuzzy logic methods and fuzzy logic structural elements for successful use within the cost model development process, and

- to propose a decision making methodology for the successful choice of fuzzy logic structural elements per fuzzy logic method corresponding to, (1) the cost model, and (2) the cost model characteristics.

## **1.8 The Origins of the Research Aims**

The effects of global competition and the advent of new technology, has meant there should be an increase in the number of cost models produced, and with minimal or a fundamentally different type of data. The aerospace sector has recently experienced the advent of new materials and new processes, leading to research projects, such as the Affordable Manufacture of Composites (AMCAPS) at BAE Systems and De Montfort University. New materials and new technologies were found to be difficult to implement commercially because of the lack of accurate cost models, principally through lack of data. In addition the development process of cost models themselves was found to be a significant cause of errors (Stockton et al 1998, Stockton et al 2000, Wang 2000). Fuzzy logic is a mathematical method for modelling uncertainty caused by imprecision, vagueness and ambiguity. Fuzzy logic is a potential method for modelling new processes with minimal data. Fuzzy logic has been used on isolated occasions for providing costs and times of products and processes (Appendix A2). It is therefore essential to identify the efficacy of fuzzy logic for cost model development over a range of possible fuzzy logic structures and cost model types. It is also essential to deliver a systematic process in which to deliver an accurate cost model using fuzzy logic, in order to eliminate errors on cost models through erroneous and incoherent cost model development processes.



## 1.9 Chapters Overview

Chapter 1 explains how the manufacturing environment is evolving towards increasing numbers of new materials and processes to satisfy a more competitive market place for new products; and how this evolution has a corresponding effect on cost model development processes. Existing work in cost modelling and cost estimating illuminates current research. Subsequently the need for a new cost model development method, fuzzy logic, is identified and justified. Finally the research aims and objectives are formally stated.

Chapter 2 introduces the cost model development process through the data identification, data collection, and data analysis methods and the cost model characteristics. Assumptions and the use of subjective judgement are examined in this context. A detailed list of requirements is identified for a new data analysis method that is fulfilled by the fuzzy logic method.

Chapter 3 reviews the fuzzy logic method and identifies a popular sample of methods. In particular the Mamdani method, Takagi Sugeno Kang method, a subtractive clustering based method and Adaptive-Neuro Fuzzy Inference System methods are studied. The difference is drawn between fuzzy logic methods, fuzzy logic structural elements that comprise the methods, and the process of using Fuzzy Inference Systems. Fuzzy logic is reviewed in the light of these 3 structural paradigms, and also in the context of the cost model characteristics. The structure of fuzzy logic and the cost model characteristics are key in developing a proposed decision making process for cost engineers using fuzzy logic in later chapters.

Chapter 4 develops an experimental plan for testing the fuzzy logic structure on a non-linear and linear model form. Such forms are deemed as comparable to the range of typical cost model forms. In particular the Taguchi methodology is used to build a range of fuzzy logic cost models from the different methods and possible different fuzzy logic structural elements. The numerical experiments include a comparison of the fuzzy logic method to the Multiple Linear Regression method.

Chapter 5 reports the results of the numerical experiments, and draws attention to key trends and points. The notion of a “best structure” and “best model” is formed, explained by the presence of interactions between fuzzy logic structural elements within the same Fuzzy Inference System.

Chapter 6 draws together the key concepts of the thesis to form a proposed decision making process for cost engineers to use fuzzy logic. The cost model characteristics, fuzzy logic structure, universal function approximation concerns, previous cases from the literature, rule reduction methods, and the numerical results from the non-linear and linear model forms are these key concepts. In addition Chapter 6 develops knowledge about fuzzy logic that can be used by cost engineers to develop a corresponding understanding and appreciation of this new field.

Chapter 7 draws the conclusions of the research, including the best results from the numerical experiments. Chapter 8 lays the ground for further research

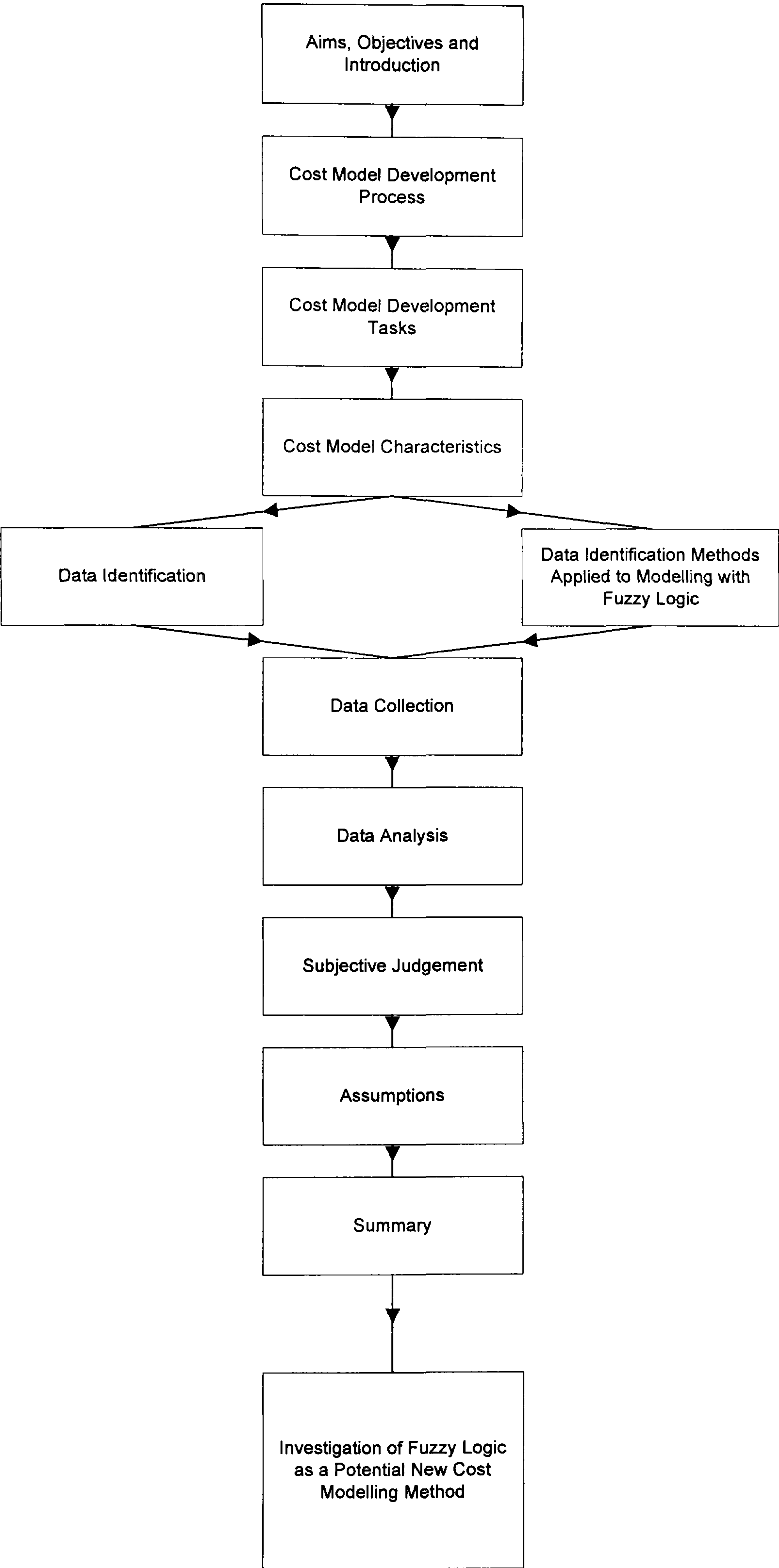


Appendix A1 holds important mathematical theory about fuzzy logic. Appendix A2 presents a sample of the potential case base available from the literature, including issues from the cost model characteristics, and structure developed in Chapter 6.

The following research uses the Fuzzy Logic Toolbox for MATLAB 6.1 from the MathWorks.

**CHAPTER TWO**

**THE COST MODEL DEVELOPMENT PROCESS**

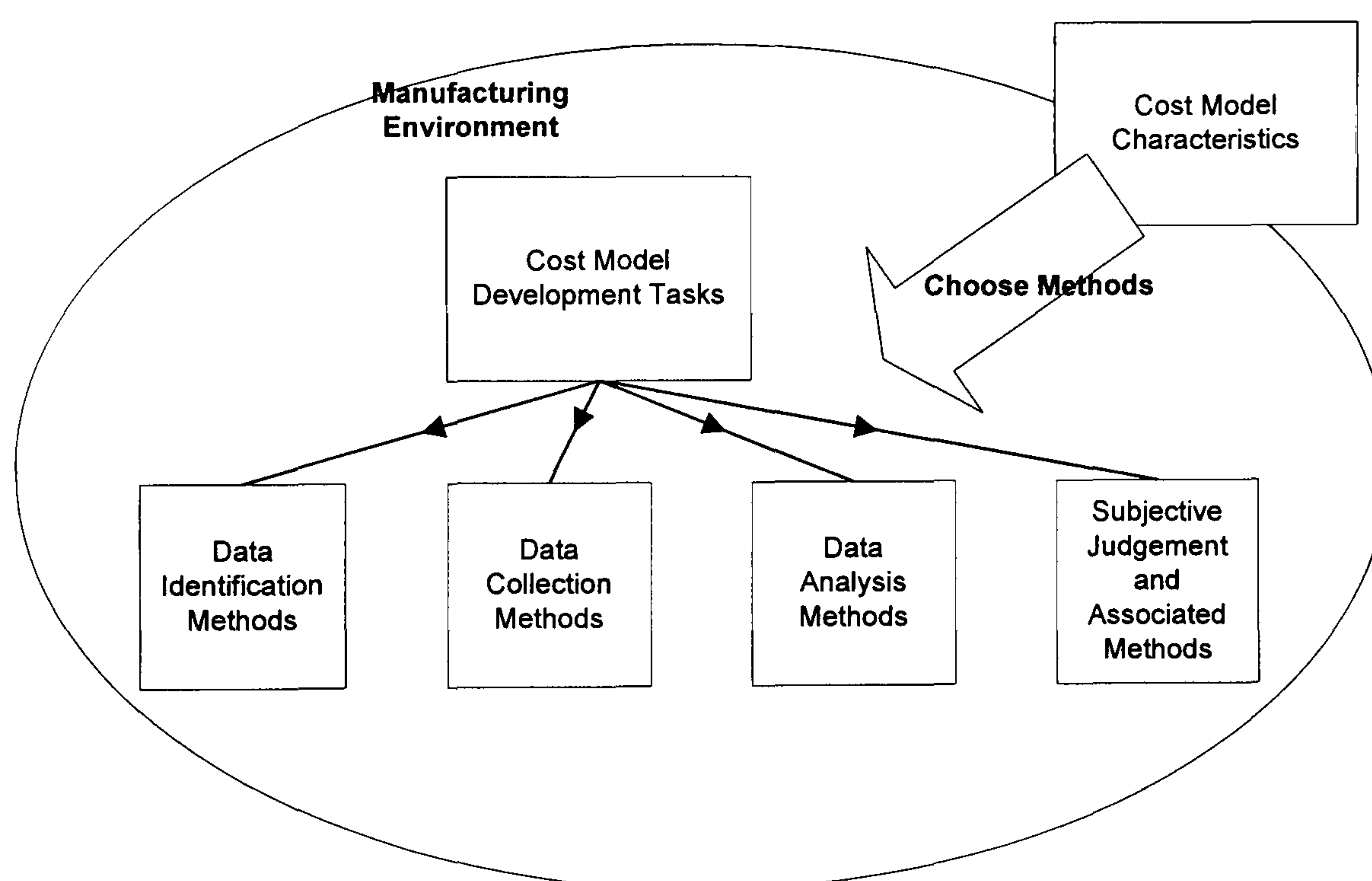




## 2.1 Introduction to Chapter 2

Chapter 2 further examines cost modelling from the point of view of the cost model development process as touched upon in Chapter 1, associated cost model development tasks, and a defining set of cost model characteristics. The cost model development process is further categorised into Data Identification (DI), Data Collection (DC) and Data Analysis (DA) tasks and methods. The cost model characteristics are described and introduced as a path towards selecting methods to carry out these cost model development tasks. The research structure is shown in Figure 2.1.

**Figure 2.1: Structure of the Research.**

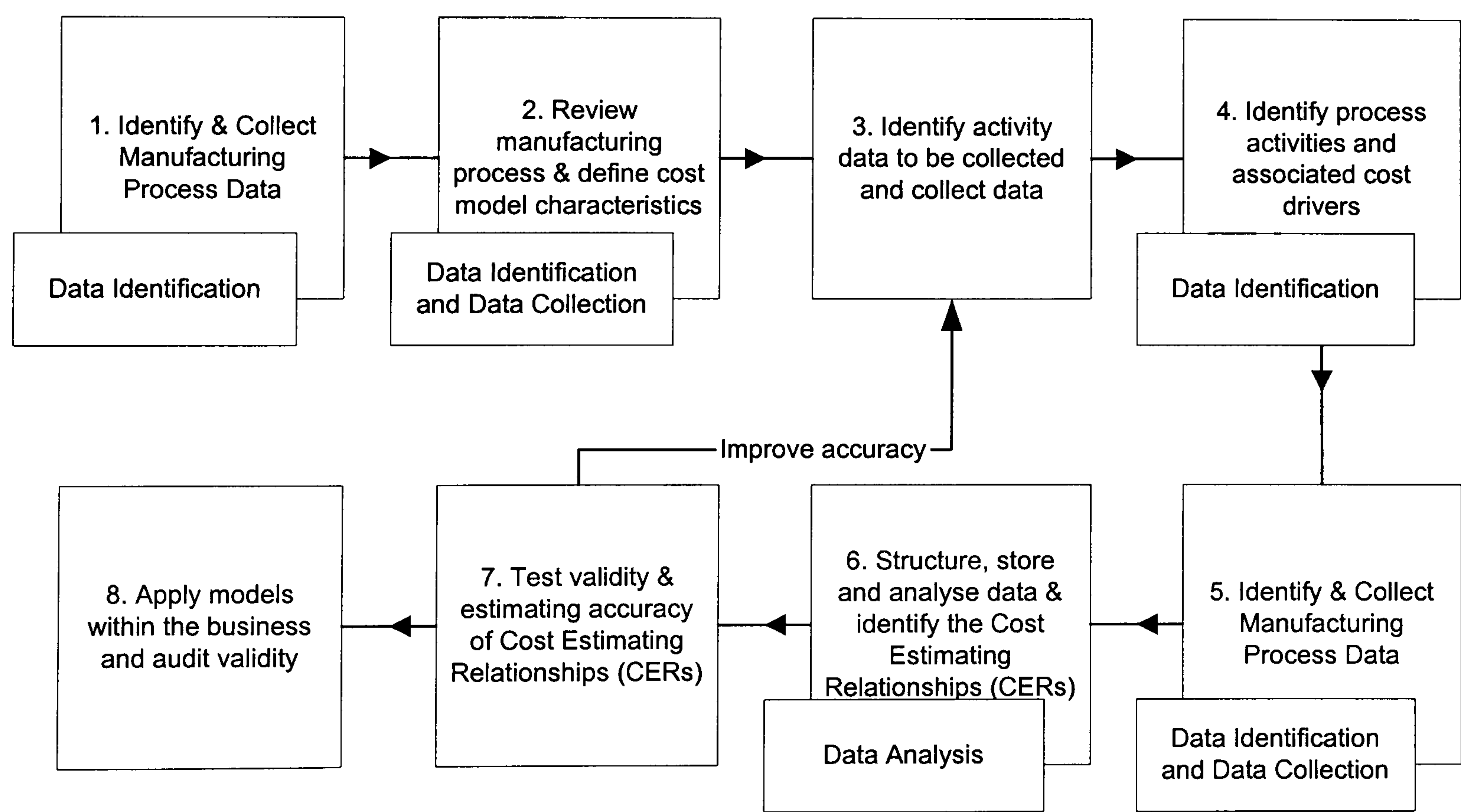


## 2.2 Cost Model Development Tasks

There are a number of basic tasks within the cost model development process. These are related to the cost model characteristics. The cost model development process is the collection of tasks required to build a relationship between costs and predictor variables. Stockton et al (1998) used a cost model development process that was later adapted by

Wang (Wang 2000). This later cost model development process was represented as a series of tasks as shown in Figure 2.2. Three general functions of these tasks are (1) data identification, (2) data collection and (3) data analysis. Without any formal model such as Figure 2.2 the cost model development process becomes ill defined, ad hoc and a source of error within individual cost estimates (Lederer and Prasad 1993). This situation was found within industry, i.e. that formal methods were lacking in many areas of the cost model development process, and indeed that subjective process expertise was heavily relied upon in order to generate a cost model, (Rush, Roy and Tuer 2002, Stockton et al 1998, Layer et al 2002).

**Figure 2.2: Cost Model Development Process (Wang 2000).**





### 2.3 Cost Model Characteristics

A change in one cost model characteristic influences the development of cost models. Stockton (1983) identified the cost model characteristics as shown in Table 2.1. The specific methods used during the development of a cost model are highly dependant on these characteristics. How the cost model characteristics can affect the choice of cost modelling methods are influenced via the issues in Table 2.1.

**Table 2.1: Cost Model Characteristics and Issues.**

<b>Cost Model Characteristics</b>	<b>Issues</b>
Manufacturing volumes	Customer demand, cumulative errors, data availability
Variety of tasks	Repeatability of measurements, process planning
Repetitiveness of tasks	Amount of historical data, allowances made for worker fatigue
Accuracy	Choice of methods, amount of data, level of decision making
Amount of subjective judgement	Expert choice, age of technology
Personnel whom operate the system	Usability, repeatability, understandability
Detail of input data	Cost of data collection related to detail, location of data
Estimate application time	Data availability, data pre-processing, workflow
Operating costs	Frequency of use, decision making level
System set-up costs	Level of technology innovation, cost data demand

The model identification process is subjective, inconsistent, poorly documented, black box, unstructured and individualistic. The model identification process involves identifying data that affects cost and putting this data into the form of a function. The form might be decided by subjective engineering judgement or by experience and statistical grounds. For example, many cost models are initially in the form of power

laws with parameters that must be determined experimentally built on this assumption (Dhillon 1989, Thuesen and Fabrycky 1989).

Method selection is an important choice to influence cost model characteristics. Attempts have been made to define the level of a cost model based on its estimating accuracy, so that this research has attributed names to Average Percentage Error (APE). These are from the American Association of Cost Engineers (AACE) (Remer et al 1996, Humphreys 1991):

- (a) -5 to +15% APE described as definitive
- (b) -15 to +30% APE described as budget
- (c) -30 to +50% APE described as Rough Order of Magnitude (ROM)

## **2.4 Data Identification**

Seo et al (2002) identified methods of identifying data, i.e. the data was described either by “yes” or “no”, or the data was ranked or the data was estimated or “specified”. Wang found (Wang 2000, Stockton and Wang 1999) that the data identification tasks must be able to:

- a) identify data that relates to cost from a variety of data sources,
- b) identify data for new processes that have no historical data, and
- c) have the potential for identifying cost drivers from non-cost driver variables.

Rush and Roy (2001a) used process mapping and related understanding in an integrated product team environment to identify cost drivers for the product design activity.



Seo et al (2002) use process experts and a literature review with a list of Life Cycle Cost factors to identify data or product attributes. Other methods of deriving product attributes included:

- a) eco checklist (Fiksel 1996), and
- b) design parameters (Alting and Legarth 1995)

A sample question from the eco checklist for identifying Life Cycle Cost was: “what type of energy is required when using the product?”. The design parameters in (b), e.g. engine power, were linked to life cycle parameters, e.g. mass or efficiency. They were linked to each other using a literature review and process experts. The method employed by Seo et al (2002) involves more subjectivity as it was for a high level model, than a method employed by Jiao and Tseng (1999), whose approach relies on extensive historical data for previous product variants.

When experts prepare cost estimates the steps are difficult to follow from one case to another and can lead to variations in cost simply because of this inconsistency within the cost model development process (Rush and Roy 2001a, Lederer and Prasad 1993, Stockton et al 1998). Walter (Walter 1997) cites poor communication as the reason for inconsistency when a cost estimator uses an engineer to cost a design. Methods used to identify data, i.e. derive parameters, are listed as:

- (a) rules of thumb,
- (b) expert knowledge, and
- (a) historical calibrations.

A difficult problem faced by cost modellers is how to build a model when a large set of variables has been identified as affecting cost. One principle used to circumvent the problem is the pareto principle or its effect. The pareto effect is that 20% of variables produce 80% of the cost. Gutowski (1997) explains how the pareto effect is apparent particularly in hand lay-up and assembly processes. Hence the pareto effect greatly reduces data identification and collection effort.

Data identification may involve identifying abstracted data. For example Dean and Unal (Dean and Unal 1991) examined the design for cost from a parametric costing perspective. They refuted belief that reducing weight led to reduced costs and discussed that a complexity term in their model, derived from a database of historical data, was the main cost driver. This “complexity” can therefore be considered as abstracted from data. They further discussed that reducing complexity led to a reduction in costs and drew attention to the system that produced the product and how costs were generated from it. Complexity can be used to aggregate the influence of many variables into one. Boothroyd and Dewhurst (1985) use a subjective measure of complexity in their work on Design For Manufacture and Assembly (DFMA). DFMA very much has cost as a concern. In fact Boothroyd et al (2002) describe a reduction in parts, and hence complexity, as effectively reducing costs of assembly and other overall costs.

Shehab and Abdalla (2001) noted the use of features and cutting conditions for material selection in the proprietary software Cambridge Material Selection (CMS). Their overall system involves a constraint of automatic feature identification using their



Computer Aided Design (CAD) system. The alternative might be the varying success of feature identification by manufacturing or design engineers as experts.

Stockton et al (1998) were successful in identifying data for a process time model for automated tape laying. The times were used to decide on the capital outlay for machines that cost of the order of millions of dollars. The model successfully reduced capital outlay for a reduced number of machines.

Data identification can be affected by data sample size. A disadvantage of the Activity Based Costing approach is that a cost element may be large, so that if it changes due to its random nature, large variations in cost will result (Asiedu, Besant and Gu 2000). Such variables must be accompanied with a large data sample to effectively predict their effect on cost.

Mileham et al (Mileham et al 1993) developed an automated system for parametric costing of a set of plastic components, which used three main data sources in its formulation

- (a) “The basic information available to the designer at the conceptual design stage,
- (b) a database of component information that enables cost-parameter relationships to be assessed. This data needs to be process specific, and
- (c) a generic set of component parameters that is capable of describing component characteristics.”

The data sources therefore directed the data identification task that, in this case, involved choosing parameters that affect cost.

Colmer et al (1999) state the importance of experts from other fields but the same technology if this technology is new, in identifying data. Their importance is in experience with material quality and deposition rates, or material strength for instance.

## **2.5 Data Collection**

Important common data collection methods were found to be: Maynard Operations Sequence Technique (MOST), Methods Time Measurement (MTM), IDEF process charting, video tape recording and team “brain storming” (EPSRC grant GR/M/58818/01 deliverables one).

Choi and Ip (1999) compared Methods Time Measurement, (MTM), and Robot Time and Motion, (RTM) for directing the data collection effort. Important differences were found between the two methods that reflected the difference between robot motion and human nature within work. MOST, MTM and RTM require tasks to be broken down into their basic elements in order to estimate the time for a complex task such as assembly. Assumptions concerning the nature of robot and human tasks need to be considered when estimating times, (e.g. the number of parts being used in assembly, velocity and acceleration elements and the weight of components under assembly). Such assumptions were used to effectively differentiate between the two.

Data collection is time consuming and constrained by the available data sources. For example the data collection method of time study is time consuming in several respects (Currie 1977). The industrial engineer collecting the data must be familiar with the work taking place. In addition the work force must be prepared to take part in the study



to eliminate unnecessary variation in times. Finally the study must be made over a large number of instances and at varying times for a statistically meaningful sample to be taken, and allowances for the environment to be accurately applied. The workforce, sample size and conditions thus constrain data collection.

Data collection is time consuming when a large amount of data is required. Different versions of MTM attempt to simplify the process by using less work elements or some versions are intended for specialised situations, e.g. MTM 3 for small batch work. Maynard Operation Sequence Technique (MOST) is a further simplification of MTM 1 with the aim of maintaining the same accuracy. The simplification is attained by collecting common sequences of elements that occur into one element for rapid application. Zandin et al (1990) stated improvements that amounted to 3 times the speed of MTM systems when measuring work using MOST. Cohen et al (1998) use Automatic Speech Recognition (ASR) to improve the process of time measurement when generating work measurement time standards for MOST by a time reduction of 70%. Data collection is therefore simplified and made less time consuming by using technology.

Luong and Spedding (1995) use a knowledge based system to reduce estimation time by integrating cost model outputs with process planning and machinability, using design rules, and allowing for instantaneous access to data (labour rate, machine tool rate, cutting speed and Brinell hardness) for inexperienced workers. The system, therefore, involved computerisation and rationalisation of data sources for instantaneous access to data.

The relationships produced by the cost model development process are termed Cost Estimating Relationships (CERs). The parameters are termed attributes of the product or process. The most important aspect of parameters is that they are measurable (Dean 1989). Similar products or systems are used in order to link the measurable attributes to cost. If a Cost Estimating Relationship is found then the variables affecting cost are termed cost drivers. Dean (Dean 1989) describes cost drivers as those attributes that define the requirements of the design.

Nwagboso and Whitehouse (1991) in using a literature review found that direct observation of tasks and robots had been used when estimating robot work cycle times. They also identified RTM or robot time and motion that uses tables, regression, kinematic equations and path geometry to determine robot cycle time. Other methods include computer-based simulation of specific robots, which has the advantage of using knowledge about them while not specifically stating where and how this knowledge was obtained. An example of this 'knowledge' was in estimating the cycle time for assembly using an IBM 7535 SCARA type robot (Mayer and Jayaraman 1983). Here robot motion in terms of velocity of travel is not altered by changes in payload.

Han et al (2001) state the problem of data collection of tolerances from CAD solid models that are only recorded as text and not in data structures for easy migration to downstream processes. Hence this data has to be processed manually. Han et al (2001) also discussed feature identification and interpretation resulting in a collection of different interpretations of features for the same product that have different manufacturing costs. These features are interpreted as geometric or via different



manufacturability criteria e.g. process planning. It is these “downstream” criteria that facilitate feature identification that must be optimised in terms of cost. Optimisation is difficult, i.e. an example of 20 pockets in a component led to 1296 interpretations as features.

Despite the absence of detailed data “design for cost” systems have been built for the conceptual design stage (Shehab and Abdala 2002, Ou-Yang and Lin 1997). The Cost Optimization Software for Transport Aircraft Design Evaluation (COSTADE) (Mabson et al 1996) uses a variety of individual cost models for estimating a range of cost types, hence providing flexibility, at the design stage.

The nature of the data collection task is likely to change with the increasing use of new materials and processes, i.e. due to the absence of readily available historical data then the data collected may move more towards qualitative data and the use of linguistic variables. Such types of data, information or knowledge will require novel data collection methods. Rush and Roy (2001b), developed a tool for ‘rationale capture’ to collect data about the assumptions and reasoning made when cost modelling. They used a novice to document the tasks made by an expert to better capture his reasoning by eliminating tendencies towards missing important steps out.

There is a need to minimize the data collection process i.e. obtain a required accuracy in the least cost and time manner. Dean (1995) noted that a leading producer of military aircraft was considering the possibility of discontinuing data collection, due to the added cost. He emphasized that cutting data collection, though being less expensive,

compromised an ability to control project costs. The result was an even greater incurred expense.

Jiao and Tseng (1999) found difficulty with product costing due to data collection for the large array of information sources and populating cost structures. Within the aerospace industry, applications have specific elements of cost arranged typically within a hierarchy into which calculated cost data is placed. These hierarchies occur by function, organisation, by product feature or by work breakdown structure (Sheldon et al 1991). Activity Based Costing (ABC) is an accountancy-based method that introduces a cost structure that seeks to more fully explain overhead costs. The older method of using direct and indirect costs was distorting the situation as increased automation decreased variable direct labour for the expense of capital overhead. When overhead is not considered it would seem that large savings have been made. ABC is costly and not straightforward to implement into a company.

A particular solution to minimising data collection is to include centralising data for reuse in a database. Pham and Ji (1999) use a selection criteria (materials to be cut, machining types, tolerances and surface finishes, and feature dimensions) to search and identify data from many alternatives of cutting tools that have been collected and stored in a “dbaseIV” data base. Machining times are subsequently calculated using feed rate and cutting speed of the selected tool.

Luong and Spedding (1995) argue that detailed process planning is required to provide data from which to generate cost estimates. They also argue that, both the cost



estimation and process planning function would be aided by computerisation as both are labour intensive in terms of the large number of tedious calculations associated with them. For example, Ou-Yang and Lin (1997) have examined the ability to generate cost estimates over the life of the injection moulding process. They identified that estimates made after the computer-aided design of the mould is completed are based on less complex data (e.g volume or mass) than when estimates are made after mould construction, i.e. when actual detailed data is available.

Data collection is constrained by the requirement for consistency. Maynard (1971) described the use of standard data sets when building a cost model, as standard data sets improve consistency when making cost estimates. Such standards were particularly appropriate for use during lifecycle costing, for example the Redstar database at NASA is an organisation wide tool containing costs and related technical and project data.

Stickel (1999) identified that small companies leave the cost engineering activities of estimating, cost control and forecasting as part-time. In large companies he identified four cost engineering roles as levels of abstraction, i.e.:

- a) product area (or country) cost engineering leader
- b) regional cost engineering process owner
- c) plant cost engineering contracts and,
- d) contract cost engineers.

He identified the need for cost engineers to use the same terminology when describing accuracy and adopt a standardised format with standard methods in order to improve communication.

Walter (Walter 1997) describes how cheap parametric estimating is in comparison to bottom up (savings of \$1 billion can be made in one organisation). Parametric estimating uses instances of high level data that can be as low as only 5 cost examples. Further data collection involves subjective judgement concerning the usability of CERs.

MOST was used by Jiao and Tseng (1999) in order to estimate the time required for standard routings within product costing. The standard routings (as represented by a process flow diagram) were generated from historical production documents and data and specific routings that were generalised, in order to produce a general manufacturing process plan. This was used by all products that needed the manufacture of only certain features. The aim was to introduce an intermediate Time Estimating Relationship (TER) that would later be used to build a cost estimating relationship using the cost driver attributed to the feature being manufactured. This simplified the direct apportionment of costs to cost drivers via an ABC method by now only calculating cost rates for the TER. Data collection was further minimised by making manufacturing consist of one general modular set of process activities, i.e. data had been collected only once but used more than once.



Pham and Ji (1999), Leibl et al (1999) and Han et al (2001) use a CAD model to collect data about features and geometries. Data collection is minimised by automation including data reporting.

The frequency of data collection tasks is minimised by Yoshikawa et al (1990), i.e. they describe the use of cost tables as assembling all cost information into a format that may be stored in a database, but which may also be represented within the tables as a decision tree. Cost tables are typically assembled by a team of management accountants with experience in production and procurement, for example. In Japan cost tables can also be acquired from private firms such is their need and popularity. The purpose of a cost table is to provide the relevant cost information at the right time. There are three types of cost table: approximate for design, detailed for production and another for purchasing at optimum prices. Particular applications are target costing at the design stage; cost reduction and value improvement; and choosing and maintaining products.

Integrated systems (Shehab and Abdalla 2001) can use artificial intelligence techniques to collect data, for example production rules, frames and object oriented methods for representing cost information and design rules for representing expert opinion. The data collection methods constrain the format of the data and might induce data processing in order to use them.

It is essential that data is in the right format before being input into a cost model (as indicated within the NASA Parametric Cost Estimating Handbook). An example of this is ensuring that data is put into the accounting system of the target company or product,

rather than use data within a proprietary cost estimating software package. Similar problems occur when using work breakdown structures, especially within analogical estimating. Luong and Spedding (1995), in developing a knowledge-based system that could cost estimate parts that undergo the process of hole making, stated that a major problem in the building of a machinability database is the format of the data. They chose two formats, the machinability data handbook and cutting tool suppliers' format. The machinability data went towards giving a cost estimate and when data was empirical for machinability, Lagrange interpolation was used for gaps between data points.

Collected data must be timely. The value of knowledge decreases over time and the uncertainty of knowledge increases into the future. Cost modelling is improved by methods that better interact with knowledge at key stages of the product development life cycle, i.e. times are targeted to improve effectiveness the most. Ostwald (1974) stated that the accuracy of a cost estimate is inversely proportional to its age.

Data collection can be minimised by data identification. The identification of the level of detail required, e.g. from a statement of requirements for a high or low level model (Scanlan et al 2002) directs data collection. Levels of detail are highlighted by their hierarchical structures. These hierarchies occur by function, organisation, by product feature or by work breakdown (Sheldon et al 1991).



## 2.6 Data Analysis

Data analysis examines instances of data in order to look for a relationship between them, i.e. “Cost Estimating Relationships”, CERs. Ideally the data analysis task (Wang 2000, Stockton and Wang 1999) must be able to:

- (1) cope with a large number of predictor variables,
- (2) rank the individual variables in order of their relative importance on costs,
- (3) operate without knowing the form of the cost function, and
- (4) establish the relationships between cost and the variables of each of these costs.

Pham and Ji (Pham and Ji 1999) discuss implementing their concurrent design system that produced machining times at the concept stage. The machining times are calculated from handbooks and formulae, the information for which is generated from inputs to, and automatically generated within, the designed system. Such inputs include machining operation types, cutting tool types, cutting parameters and feature geometry, feature shapes, dimensions and feature relationships, tolerances, surface finish and material property requirements. The large variety of inputs means data collection from the shop floor and manufacturing knowledge acquisition allows a more significant data analysis concurrently, i.e. CAD data is used for automatic feature recognition. Features subsequently generate the data previously collected from the shop floor, and manufacturing knowledge to calculate the times from formulae.

Subjective judgement may be employed to determine the type of relationship between the resource dependant variable (e.g. direct labour or cycle time) and the predictor

variables (e.g. process or product features). For example the wrong choice of linear regression analysis for a non-linear relationship makes for large errors. Pedrycz et al (1999) give an example of a data set from the area of software engineering relating software size to software effort or software size to time, i.e. the Yourdon 78-80 survey data. The plot is highly scattered and is eventually modelled using fuzzy-C-means clustering because of its non-linearity.

The existing data analysis methods of neural networks and Bayesian analysis, using subjective judgement, are complex, time consuming, and can be poorly understood by their users. An example of Bayesian analysis involves choosing a probability density function (pdf) to model an unknown parameter as a random variable (Weerahandi 1995). It is not straightforward or obvious in choosing the best pdf from amongst many and whether just estimating a value for this unknown parameter might be better. In addition probability is based on measures of relative frequency of occurrence of events. Hence probabilities are related to quantitative measures.

Kim and Dornfeld (Kim and Dornfeld 2001) developed a model for estimating cycle times for a drilling process in a mass production environment. A control chart for burr formation was made with respect to process parameters via experimental results. A method of Bayesian analysis was used in order to predict different types of burr formation and hence minimise the total cost of the process. The different deburring processes had already had their costs estimated and Baye's rule is used in this example as it can be updated when more data becomes available. Initially a uniform probability



distribution was assumed for the occurrence of a variable associated with deburr, i.e. its values were equally likely to occur randomly.

Case-based reasoning facilitates the choice of examples and previous cases to generate solutions. Case based reasoning is open to subjective judgement in how to classify the different cases and how similar one case is to another. A subjective algorithm may choose a similar case or it may depend on the magnitude of the difference between a case parameter value and the target parameter value. Some case based reasoning applications are based on a group technology coding scheme for categorising product or process features. It is therefore important to identify the parameter which is most important for similarity. Once a case has been chosen there may be a large amount of historical data attached to it, implying a resource intensive set up and maintenance procedure. Li et al (1997) maximise a signal to noise ratio in order to select the features that cause defective bearings. To overcome the need for using process expertise, Rush and Roy (2001b), use case based reasoning in which historical data is extrapolated to generate a cost model for a new process or product. The cost estimate or model can be modified using expert knowledge, or by changing theoretical equations to suit the new case. Jiao and Tseng (1999) refer to the “comparative” method as the group technology method because of its use in classifying similar components, often by a coding scheme. They point to the assumption that relationships between similar features are always linear.

Smith (Smith 2001) used a form of case based reasoning when he described a bottom up estimating application as deriving cost estimating relationships for systems that made up

the whole. Statistical methods were therefore applied to smaller parts of the whole. The point of the exercise was to derive a cost per unit of weight relationship for an unmanned experimental aircraft from other aircraft, by subtracting systems that were manned and hence not required.

Comparative costing is a cost estimating process that can be used to identify data for a cost model. Comparative costing is the generalization used to describe similarity based costing and analogical costing. They seek to derive a cost, or identify data from previously recorded cost estimates or previous particular models, for similar products or components. More than one analogical project can be used in which case choosing those that minimise the average error of these projects typically occurs. Case based reasoning can be used to seek these similar cases based on numerical measures or object oriented based approaches (Rehman and Guenov 1998). Numerical measures can be based on a distance metric between the input parameters for the target and other cases (Jeffrey and Walkerden 1999). Case based reasoning identifies data by:

- (a) substitution directly from the retrieved case,
- (b) subjective derivation from the retrieved case, and
- (c) transformation of the retrieved case.

Chung and Huang (2002) used queuing theory in order to estimate the cycle time for wafer fabrication using engineering lots. Engineering lots were also termed technology development lots and importantly did not constitute a full batch. Subsequently, therefore, cycle times differ when using engineering lots. They cited both simulation and statistical methods as a way of predicting process times. In the case of the latter,



short-term prediction was stated a particular strength. In order to improve long term prediction subjective judgement is required to manipulate the statistical method, e.g. “banding” a regression analysis, or make assumptions for calculations, e.g. component demand follows an assumed pattern in simulation. These methods are particularly suited to situations where more than one process is required to manufacture the part and dynamic interaction occurs between individual processes.

MOST and MTM provide data for Predetermined Motion Time Systems (PMTS) (Wild 1990). Cost estimates can be built up from these MOST and MTM elements, i.e.:

- (d) jobs are broken down into elements,
  - (e) jobs are rated subjectively by comparison with existing work standards,
  - (f) the times of different elements are totalled, and
  - (g) allowances are made for e.g. fatigue or rejects,
- i.e. MOST and MTM are used to measure work.

Subjective judgement is used to match actual work to work standards and to judge which allowances should be made. This whole process hinges on matching interpretations of elements using a simple scale. Lack of knowledge or data of elements compromises the data analysis. Because subjective judgement is all pervasive small errors total into a final large error.

The performance of the data analysis method may be impaired by a lack of data. It was found in industry that, for example in a highly complex low volume product, e.g. engines for the aerospace industry, a limited number of engines in an engine series, and

hence a limited number of costs, may be available for high level cost modelling. The stage of the product development life cycle dictates the amount of data available.

Farineau et al (Farineau et al 2001) address the problem of building models early in the design process when only “partially defined products” are available from which to extract data. Examples are provided of alternative cost modelling methods and the stage of the product lifecycle they are able to provide cost models for, i.e.

- a) parametric methods typically applied at the conceptual design stage, i.e. (1) method of scales (£/Kg) where simple ratios are used and relationships are assumed to be linear; (2) a breakdown of a product into simple statistical relationships, and (3) CERs by linear least squares best fit or an assumed power relationship and,
- b) analytical methods typically applied at the later stages of the lifecycle, i.e development and production. A significant amount of time is required to collect data from such a detailed breakdown of the process and / or product. However, the collection of detail can be assisted by computerized tools.

An important question is: what is the chance of the estimating accuracy being a certain percentage? Analytical methods are not suited to developing models at the conceptual design stage since they make use of high levels of data collection, which require a detailed breakdown structure of the product and process under consideration. Consequently cost models developed analytically at the conceptual design stage have a greater possibility of being of poor accuracy. Seo et al (Seo et al 2002) provide an example of parametric estimating to develop a model, using both regression analysis and an artificial neural network, for predicting Life Cycle Costs at the conceptual design stage. The neural network was trained using attributes from 150 products. The test



predicted LCC to be 0.11%-12.02% Average Percentage Error of the corresponding LCC analyses. In contrast parametric techniques (used because of a lack of data) can only predict LCC from existing LCC to -30% to 50% Average Percentage Error.

The importance of being able to judge the accuracy of a method is shown by Kelly et al (1995). Kelly et al (1995) point out that too much variation in a product or process may make a Multiple Linear Regression (MLR) method unsuitable. Other problems occur in the addition of new data to maintain a model being unsuitable because of inflation or changes in technology.

Kelly et al (1995) gave an example of Multiple Linear Regression (MLR) analysis used in the modelling of time for Computer Numerical Control (CNC) part programming for wing stringers that gave favourable results. MLR is promoted as a “rough cut” estimating tool at the conceptual design stage. The term rough cut suggests a departure from time and resource consuming detailed algorithms and the introduction of subjectivity to facilitate assumptions about cost drivers and support simplicity. It was therefore deemed by this method that stringer length and number of “ribs” in contact with the stringer, should form the explanatory variables for an MLR model, and was later justified numerically, i.e. a coefficient of correlation of 0.99 and a standard error of 1.9 hours at the 99% confidence level. Data from 58 stringers was used to build the model.

Cost models, when suitably developed, should enable a wider range of personnel to make cost estimates, particularly product and process designers. For example, Stockton

et al (1998) developed a process time model for automated tape laying. Times of activities, e.g. handling, were taken from the shop-floor, and algorithms used to predict actual tape laying time. The times were used to decide on the capital outlay for machines that cost of the order of millions of dollars.

Although attempts have been made to formalise the process by which cost models are developed (Farineau et al 2002 and Stockton et al 1998) this is often difficult to achieve since a cost model is typically made based on the amount and nature of the information, particularly cost estimates, and expertise available at the time the model is prepared. The above issues illustrate the subjective nature of the cost modelling process and its reliance on the use of experienced estimators, i.e. at the concept stage little data is available compared with the amount available at the end of the development cycle. In general as the product development cycle proceeds there is a decreasing amount of subjective judgement and an increasing level of accuracy.

There are a variety of data analysis methods for cost modelling. Wang (2000) used 16 variables in a regression model for the turning process and obtained Average Percentage Error between 10% and 20% that gradually improved from 150 data points to 750 data points, i.e. to eventually approximately 12%. These data points would have occurred at an advanced stage of the product development cycle.

Graves et al (1996) use regression analysis and a top down description of the product. It is often used with this level of parametric data and produces empirical relationships, (e.g. the Cost Estimating Relationships.) Disadvantages with this method include:



- (a) identifying cost drivers,
- (b) finding the effects of inflation on individual cost elements,
- (c) the need for knowledge about the final product, and
- (d) the need for high quality historical data.

Bode's (Bode 2000) experiments show that only a few dozen data points are in general sufficient to use a neural network to make a cost model.

Leibl et al (1999) give values for deviations from cost estimates for the cost modelling methods of neural networks (a mean error of 5% and a maximum deviation of 16%), cost growth function (a deviation of +/-10%), and for the method search calculation the deviation is described as greater than the cost growth function method only.

Other common data analysis methods were found to be Bayesian Analysis (Chulani et al 1999), linear programming, multiple regression analysis, genetic programming (Dolado 2001, Kim and Han 2003) and neural networks (Zhang and Fuh 1998, Shtub and Zimerman 1993). These methods can be combined sequentially to build a coherent process of cost modelling. Research within the Engineering and Physical Sciences Research Council (EPSRC) grant: "IMI: Improving the cost model development process GR/M58818/01" seeks to link these methods using rules in order to form an expert system.

Rune (Rune 1998) described a cost model whose output gave "poor quality" costs. The model used the:

- (i) Taguchi loss function to estimate poor quality costs,
- (ii) Activity Based Costing to calibrate key parameters of the loss function, and
- (iii) Quality Function Deployment to estimate intangible poor quality costs, for example customer dissatisfaction (an example concept is “perception of importance of requirements”). The result is a cost index that is used to produce probabilities of the occurrence of intangible costs.

Boothroyd and Reynolds (2002) offered no formal method for construction of CERs, using the particular tools of theoretical equations and empirical laws, for instance. Detailed approaches that build models and costs based on the use of theoretical equations or empirical laws is therefore used by Boothroyd and Reynolds (1994) and Diplaris and Sfantsilopoulos (2000) to connect process-based parameters to cutting speed or tolerances to cost, for example.

Some general rules were found for using regression analysis, i.e. it should be borne in mind that

- (a) care must be taken as relationships may be found that are incidental, and
- (b) the regression may be inadequate over a large range and can be split into an estimate of smaller ones joined together, called banding, to improve accuracy. Cost functions from Neural Networks are also only considered over typical ranges (Bode et al 1995, Mileham et al 1993).

Data analysis may proceed by identifying coefficients such as the cost per  $\text{cm}^3$  for metal removal (Han et al 2001, Boothroyd et al 2002). Boothroyd et al (2002) identified



several cost models for most manufacturing processes. Tables 2.2, 2.3 and 2.4 give a direct comparison for machining, injection moulding and powder metal processing.

**Table 2.2: Machining.**

	Data Identification (DI)	Data Collection (DC) / Data source	Data Analysis
Level of detail	Design rules	Machining data handbook, experience, e.g. feed×speed is the machined surface generation rate.	A balance reached between analysing simple metal removal by one machining process and that by several processes when estimating costs for the volume of material removed. Assumptions.
Accuracy			Not supplied

**Table 2.3: Injection Moulding Time.**

	DI	DC / Data source	DA
Level of detail	Those parameters that affect volume of the mould, experience.	Polymer suppliers, machine suppliers, material data.	Elementary use of mechanics accompanied by simplifying assumptions about polymer flow rate for instance. Cooling time is calculated with the help of a heat conduction equation. Reasoning is used about machine size and therefore available power.
Accuracy			Used for comparison of part designs allowing for simplification of the mechanical model of polymer flows.

**Table 2.4: Powder Metal Processing.**

	DI	DC / Data source	DA
Level of detail	Experimental, experience, expert rules	Suppliers, machine dimensions	Assumptions regarding material loss, for example. The building up of physical mechanical equations, expert rules
Accuracy			Not supplied

Recently methods from soft computing have been used for data analysis for cost modelling. Advanced techniques for cost modelling include neural networks and fuzzy logic (Bode 2000, Appendix A2). Wang et al (2001) applied neural networks in order to



estimate the cost of a turning process. Taguchi methods were used in order to find an optimum neural network structure that would estimate the function for the costs of the turning process. Historical data was available. Favourable accuracy (18%) in comparison to regression analysis (29%) was achieved, as well as a method for choosing structural elements of the neural network structure. Setyawati et al (2002) also use the Taguchi design of experiments to determine a best structure for a neural network to estimate building construction costs. The results were better than regression analysis but not as good as the authors expected. Neural networks are

- (a) non-parametric estimators

- (b) can approximate any continuous function, i.e. they are a universal function approximator.

Analysis can also proceed through the use of experimentation. Methods include Taguchi design of experiments and factorial analysis. Keys et al (1987) explain how a two level factorial analysis can be used to measure interactions between variables that influence cost, i.e. the input variables are not mutually independent, using an existing cost model.

Younossi et al (2002) used ordinary and stepwise regression when developing CERs for military jet engines for the RAND Corporation. They discard variables that have too few data points as having too few degrees of freedom and warn against the concurrent use of variables that indicate the same effect, i.e. are themselves correlated. In developing CERs, R squared and t statistics were used to indicate their quality. It is noted that high Root Mean Square Error can be due to uncertainty in the dependant variable.

Diplaris and Sfantsikopoulos (2000) developed models for estimating the effect on cost of dimensional tolerances. A literature review, empirical results and theoretical analysis was used to examine and produce theoretical equations about machining tolerance. Machine dependant variables and theoretical equations are then used to produce costs for particular tolerances or accuracy of the machined component. Theoretical equations should supply an ideal cost, although there are many sources of potential error ranging from the human to those that are equipment related.

## **2.7 Subjective Judgement**

Subjective judgement is the use of the engineering knowledge gained by the experience of manufacturing personnel to make cost models or cost estimates. Typically, at the concept stage, only a product's function is specified, leading to a very large search space of possibilities in realising the product (Maropoulos et al 1998). A large amount of subjective engineering judgement is needed to reduce the space or choose among alternatives using relative costs. Products with a very long life cycle rely on subjective factors to alter costs because of possible new materials or processes that may be available in the future.

Sometimes the subjective judgement allows for initial decisions, judgements or estimates to be made that are eventually borne out using actual data. Having experience in the aerospace industry leads to the conclusion that cost is correlated with weight. Actual data might bear this out for a new project as occurred in (Rush and Roy 2000).



Stockton (2000) identified the main data sources available for cost modelling, a subset of which were later identified for use in a Model identification Process (MIP) (Delgado et al 2002). The MIP uses brain storming to identify potential predictor variables for cost or time that can be process features, product features or process activities. The presence and importance of a relationship between the predictor variables is identified using a paired comparison method. The paired comparison occurs between variables via a matrix of scores provided by a panel of experts.

Subjective judgement can be captured in a knowledge-based system. Fisher and Nof (1987) built a knowledge-based expert system for the decision-making process of selecting new manufacturing facilities. They included a variety of cost models and expert advice. The knowledge-based system was built in order to include as many approaches as possible and to preclude the occurrence of misuse of cost estimation formulae that might be misinterpreted. Consequently the formulae could be used with figures that were assumed to take important concepts into consideration, for example tax or depreciation. The system hopes to advise on missing or uncertain values and promote certain cost models that are “quick and dirty” to limit a search space.

Hseigh (2001) uses expert judgement and fuzzy sets in using linguistic variables to model subjective judgement when measuring manufacturing quality in conjunction with the Taguchi method and neural networks.

The use of subjective judgement is considered sufficiently prevalent as to have led to research by Roy et al (2002) into capturing the knowledge in an effective manner for re-

use. A particular problem for experts in using their subjective judgement is being aware what their knowledge is, how they use it, and applying it consistently. This problem led to Roy et al (2002) using a novice to document expert opinion. Documentation ceased after the novice had provided understanding to such a degree as to allow him or her to accurately follow cost estimating steps provided by the expert.

Subjective judgement occurs in significant amounts. Leibl et al (1999) describe input data as geometric features and semantics for their design for cost system. The system utilises design rules and allows an increasing detail of inputs from CAD design data to further improve the accuracy of the cost in terms of uses for calculation, comparison and forecast. An increase in detail follows from the description of envelope geometry to locations and nature of fasteners, i.e. the locations and nature of fasteners means for a decrease in subjective judgement in the model. Significant amounts of subjective judgement decrease with time and there is a corresponding increase in information availability.

Even at the data analysis stage experience and subjective judgement is required to assist in knowing the difference between causal relationships and those appearing by chance when using MLR (Stockton and Middle 1982).

Ou-Yang and Lin (1997) emphasise the need for product and / or process experience to interpret design data. This is particularly so at the concept stage of the development cycle in order to extract relevant parameters and their values for cost estimation. In this



respect deviations from “actual costs” normally occur more frequently when subjective judgement is used where there is lack of detail in process or product definitions.

A data source used by Carter (1990) in estimating robot assembly cycle times is the “Design for Robotic Assembly” handbook (Boothroyd and Dewhurst 1985). The handbook makes use of industrial experience to identify rules for robotic assembly. There are no apparent ways of dictating how to methodically form rules, even though the rules provide a process of improving designs in regard to high assembly cost. For example, a non-chamfered part with a close tolerance, means insertion into a hole needs an extra point in the robot assembly program to prevent collision. This point means more time and hence more cost. It is difficult to identify the relative importance of these individual rules so it is therefore difficult to use this information to rank cost drivers. Similar to the idea of reducing cost by rules, design improvement strategies were used by Seo et al (2002) in the early phases of identifying product attributes that were finally linked to life cycle cost factors.

Even though there are no formal methods of capturing subjective judgement, in contrast there are a number of frameworks through which subjective judgement might be applied. Seo et al (Seo et al 2002) primarily used a criteria list to derive product attributes for factors used in lifecycle costing. The factors contribute to the total Life Cycle Cost (LCC). The list selected attributes so that they must be:

- a) statistically linked to the factors (i.e. through correlation tests to 95% statistical significance making the judgemental assumptions that relationships between attributes and factors are linear),

- b) logically linked to the factors through qualitative criteria, e.g. material is linked to LCC factors through “extraction and processing”,
- c) easily collected during conceptual design,
- d) sufficiently different as to be used to differentiate between concepts,
- e) easily comprehended by designers, and
- f) able to cover the scope of the product life cycle.

Aguirre and Raucent (1994) developed a model for the determination of the manufacturing costs involved in wire harness assembly. Subjective judgement was used to describe the process in wire harness assembly and evolve laws and equations that describe them sufficiently. A simple equation is used to initially relate cost to total time taken including set-up, labour rates and equipment costs, component types and time needed for assembly. This assembly process was modelled using subjective judgement and experimental analysis and involved identifying physical laws. The reasoning processes in general typically involved a description of the process or physical laws about the process, (e.g. cutting). Subjective judgement was used to provide a measure of harness complexity. Other researchers have discussed the concept of complexity in cost models. For example Meisl (1988) noted the Rockwell International Technical Complexity Analysis used to predict subsystem development engineering man-hours. Dean (1989) drew attention to complexity as being more important a cost driver than weight in the aerospace industry. Jiao and Tseng (1999) discussed complexity as a cost driver in their literature review regarding product costing.



Chan et al (2002) use fuzzy logic to capture the imprecision within judgements made about quality. Klir and Folger (1992) explain how membership functions are built using subjective opinion and provide fuzzy observation channels as an example.

Subjective judgement is used to alter an accurate low level model to take into account human factors in work study. Brown (1994) collected examples of MTM elements while attempting to produce a system that adjusts for accuracy and speed of implementation. These were essentially “get put” motions and included, “Get Weight, Put Weight, Regrasp, Apply Pressure, Eye Action, Crank, Step, Foot Motion and Bend (split into Bend Down and Arise from Bend)”. Application of MTM improves with experience. Work is more easily categorised and experience increases the frequency of applying accurate allowances for fatigue or training; i.e. aggregated MTM data for aggregated activities are adjusted via a percentage factor for a change in work conditions (Currie 1977).

Subjective judgement is used for simplifying a complex system by using indices. Rajan and Roylance (2000) use a mathematical model in order to estimate the direct costs of machine breakdown. A factor and several indices are used to change the direct cost based on a set of simple rules. An example was the process index that measured subjectively on a scale of 1 to 10 how much value had been added to the product when the process breaks down, i.e. starting at 1 for machines that use raw material. These subjective indices used a data source, i.e. subjective opinion from experts, that was less consuming of time and resources to collect and use but made for simplification and variation in accuracy. A different set of indices was reported by Somerfield (1999) for a

different application, namely the costing of equipment and plant. A ratio of a current index to another is used to cost equipment based on historical data. One of the factors attributed to the change in the indices was inflation.

Subjective judgement proved successful in improving cost estimating using rules of thumb. Lederer and Prasad (1993) use guidelines to influence data identification and for improving cost estimates, e.g. in a lengthy list they identify an: “inability to anticipate skills of project team members” as a cost driver when estimating costs. Such guidelines provide focus and consistency for projects.

Rune (1998) was successful in identifying intangible costs of poor quality by utilising subjective judgement in the Quality Function Deployment framework.

Bashir and Thomson (2001) used a literature search technique to develop a parametric method for estimating effort within design. The literature search identified the factors having the main effect on variation within the design effort, i.e. the resources required to complete a design. Difficulty was reported in finding consistency in the data sets that allowed the measurement of parameters. Indeed, the identification of parameters for measuring a factor was not straightforward. For example, the size of a project was thought to be measured by the number of parts, but decreasing the number of parts led to an increase in project complexity which in turn led to an increase in design effort. The role of the cost engineer and cost estimator are studied by Rush and Roy (2001b). In relation to cost modelling Rush and Roy (2001b) describe the activities of a cost estimator as being involved in collecting data from engineers, statisticians, computer



scientists, mathematicians, economists, sales persons and accountants. A cost engineer is involved in studying designs, for example, using cost in an objective function. A difference in the objective function leads to a different optimum design.

Subjective judgement is important for visualising costs and cost driver effects. Seo et al (2002) advocate the use of a scatter plot diagram to visualise the effect of a qualitative variable on cost. Subjective opinion can be used to supply relative magnitudes to be plotted on a scale. In this way visualization is facilitated.

Subjective judgement introduces compromise by making rating scales too simple. Data collection can proceed by identifying the presence of relationships between variables or providing a number between 1 and 10 to indicate the strength of a relationship. There is no indication of the cognitive processes or the data source that caused the subjective opinion to be a yes, no or a particular number. Meisl (1988) discusses the use of interviews to capture subjective opinion and describes a cognitive process in which unrelated knowledge and facts become associated with each other over time in the mind of an expert. The inappropriate associations mean erroneous data collection when using experts over time.

Subjective judgement introduces compromise. Such compromise can occur through making decisions on a simple rating scale. The rating scale, or a simple plus or minus indicating an effect or no effect on cost, is necessary because of the large error intervals

in estimating costs. Previous judgement is required in making a decision in (1) what similar projects to use (2) how a change in a product might lead to a change in cost.

## **2.8 Assumptions**

Any method must clearly show assumptions and the range of validity of the model. Rush and Roy (2001b) further comment on the cost model development process by studying the use of expert knowledge in building models for cost estimating within a concurrent engineering environment. They argue that experts find it difficult to express their knowledge effectively and propose using a novice to question the expert in order to document expert knowledge in making a cost estimate with assumptions. Assumptions are often an essential part of applying cost estimating and arguably attempt to put into words the conditions under which the cost model is valid. They also influence the eventual calculation and understanding of the cost estimate. The assumptions should give an indication as to the effect on the accuracy of the cost estimate, especially where assumptions may no longer remain valid. Making and recording assumptions are, therefore, an important part of the cost model development process.

Assumptions within the cost model may be concerned with the values of the input parameters into the model. Here Asiedu et al (2000) explain that these parameters have uncertain values and that it is disadvantageous to take the average of a set of values that are then input into a model. The fact that an average has been taken has served as an assumption and will, therefore, influence the final estimate. In this case then the input



parameters, as well as the output parameters, have been assumed to be random variables with probability distributions associated with them.

There are also assumptions about the data that is collected and which will be used to build the cost model. Bashir and Thomson (2001) note examples of assumptions made about data they collected from two companies to build parametric models to predict design effort. These include “the projects enjoyed good management, i.e., the amount of non-productive time was small” and “there were no substantial changes in the requirements after the feasibility study.” These assumptions relate to the activity being estimated and how it was conducted.

Boothroyd et al (2002) assume at the concept design stage that costs for unnecessary operations will be eliminated and economic conditions are reasonable.

Dhillon (1989) provides a vast number of cost models for life cycle costing. The collection is difficult to comprehend as there is no explicit explanation of how they were developed and what assumptions their form were based upon.

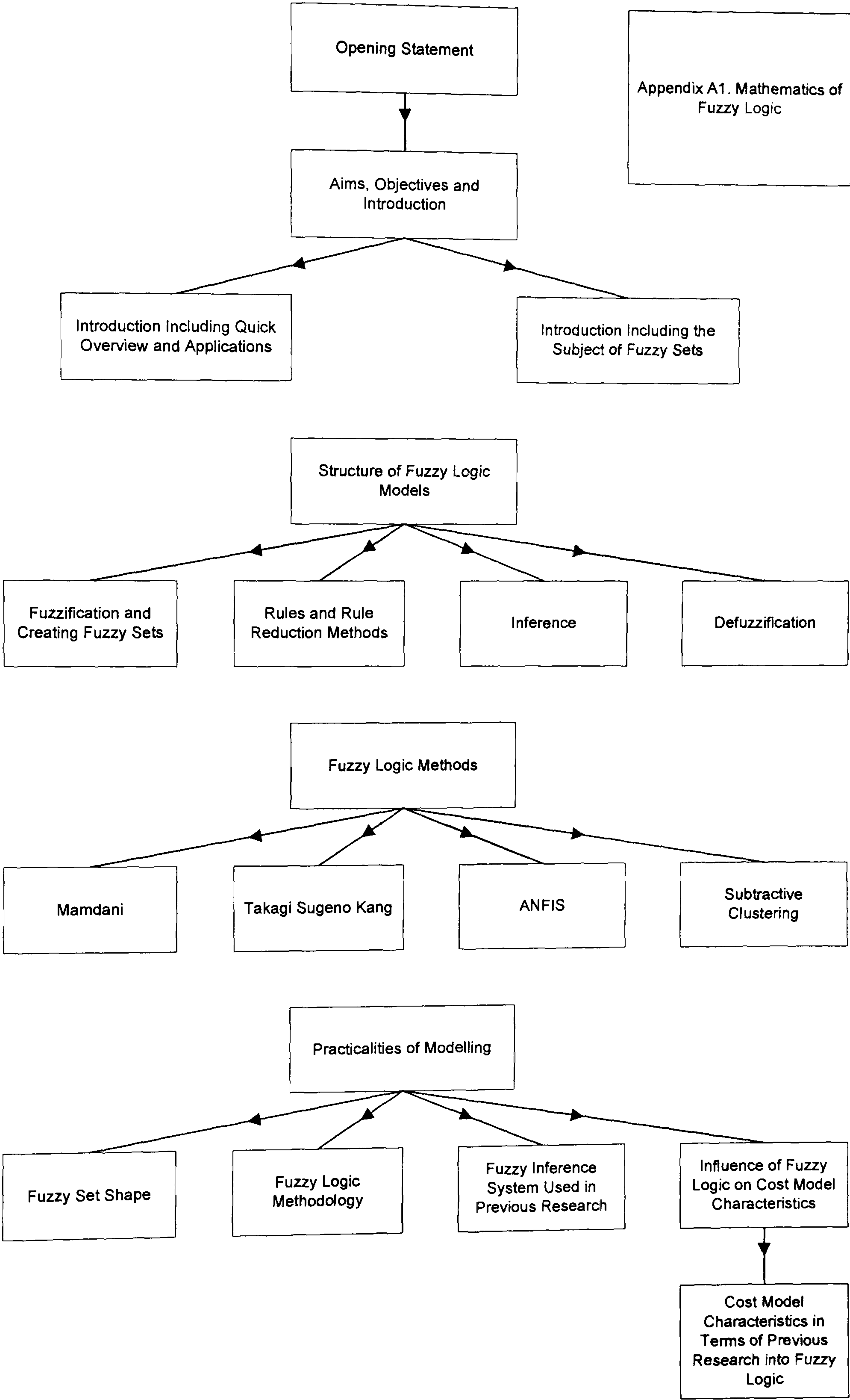
Data sources can vary in terms of their availability and the cost of the collection of data from them. If a value of a cost model characteristic, (e.g. operating costs), needs to be low, then the selection of the effective methods with which to build the cost model has been made. That is, the selection is such that only methods that can collect data that is not expensive, are considered. Similar methods that make use of low cost data are considered for data identification or data analysis. A potential disadvantage of low cost

data collection is that it may mean less data points. Bashir and Thomson (2001) use only one independent variable in building their parametric model when there are only five data points. In addition regression analysis was used in conjunction with the jack-knife method. The jack-knife method was selected based on characteristics of the data set, (bias due to its small size). A further example where data collection was decreased was found in a literature survey by Sanchez et al (1998), where the relationship between cost and tolerance was assumed to follow a hyperbolic characteristic rather than looking for a statistical relationship. The assumption made for data reduction.



CHAPTER THREE

FUZZY LOGIC



*Mamdani (1993) states: “In effect given only the actions associated with the centre point of overlapping rectangular regions of the state-space, fuzzy logic algorithm provides the action for any individual point in the state-space. Obviously all the other values are interpolated from the available finite sets of values by a method which depends on the choice of the mathematical expression used for the various fuzzy logic connectives as well as the choice of the defuzzification method”.*

### **3.1 Introduction**

Chapter 2 has reviewed the cost model development process. Cost model development consists of data identification, data collection and data analysis tasks. There is an advantage in using subjective judgement and modelling uncertainty in cost modelling for the purposes of data reduction. There are 2 broad areas of quantifying subjective judgement and uncertainty, i.e. probabilistic and fuzzy. Fuzzy logic is chosen because of its utility when there is a lack of historical data for new manufacturing processes.

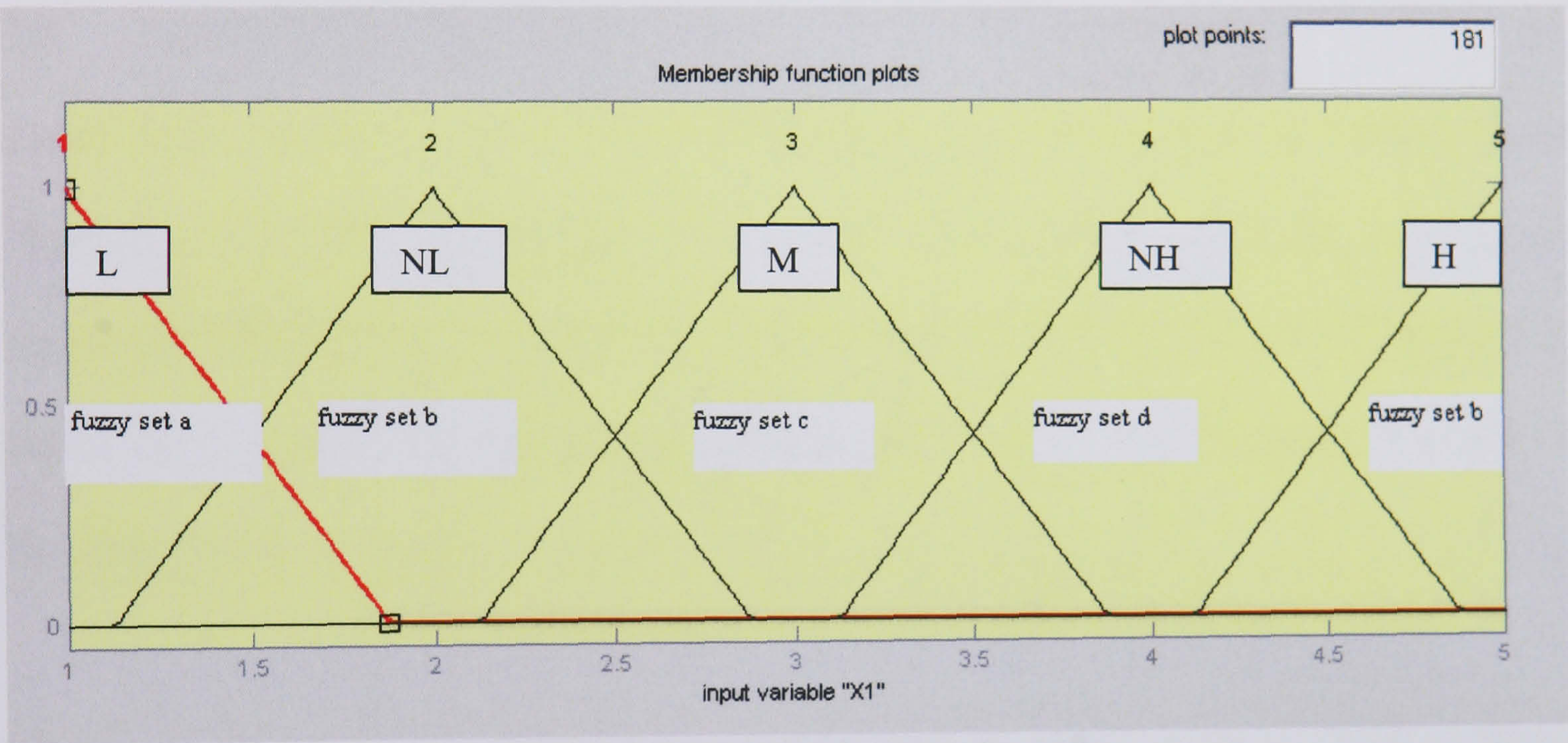
### **3.2 Fuzzy Logic Basics.**

Fuzzy logic is a mathematical technique that enables such concepts as *imprecision* to be modelled. Initially developed by Zadeh (Zadeh 1965) fuzzy logic is particularly used to model uncertainty using fuzzy sets (Zadeh 1989). The modelling makes use of parameter values to define fuzzy sets (for example in Figures 3.1 and 3.2). The fuzzy sets are themselves subsequently defined using linguistic “terms” (e.g. low, medium or high) that describe a linguistic “variable” (e.g. height or weight). These “terms” of variables are then subsequently employed within fuzzy “if-then” rules as subjective or qualitative modelling. The concept of *imprecision* can be thought of as a collection of objects that lack well defined boundaries (e.g. “the class of systems which are approximately equivalent to a specified system”) (Zadeh 1996). Fuzzy set theory also models such concepts as “vagueness” and “ambiguity”. Here Roychowdhury and Pedrycz (2001) describe vagueness as being due to a lack of complete knowledge and



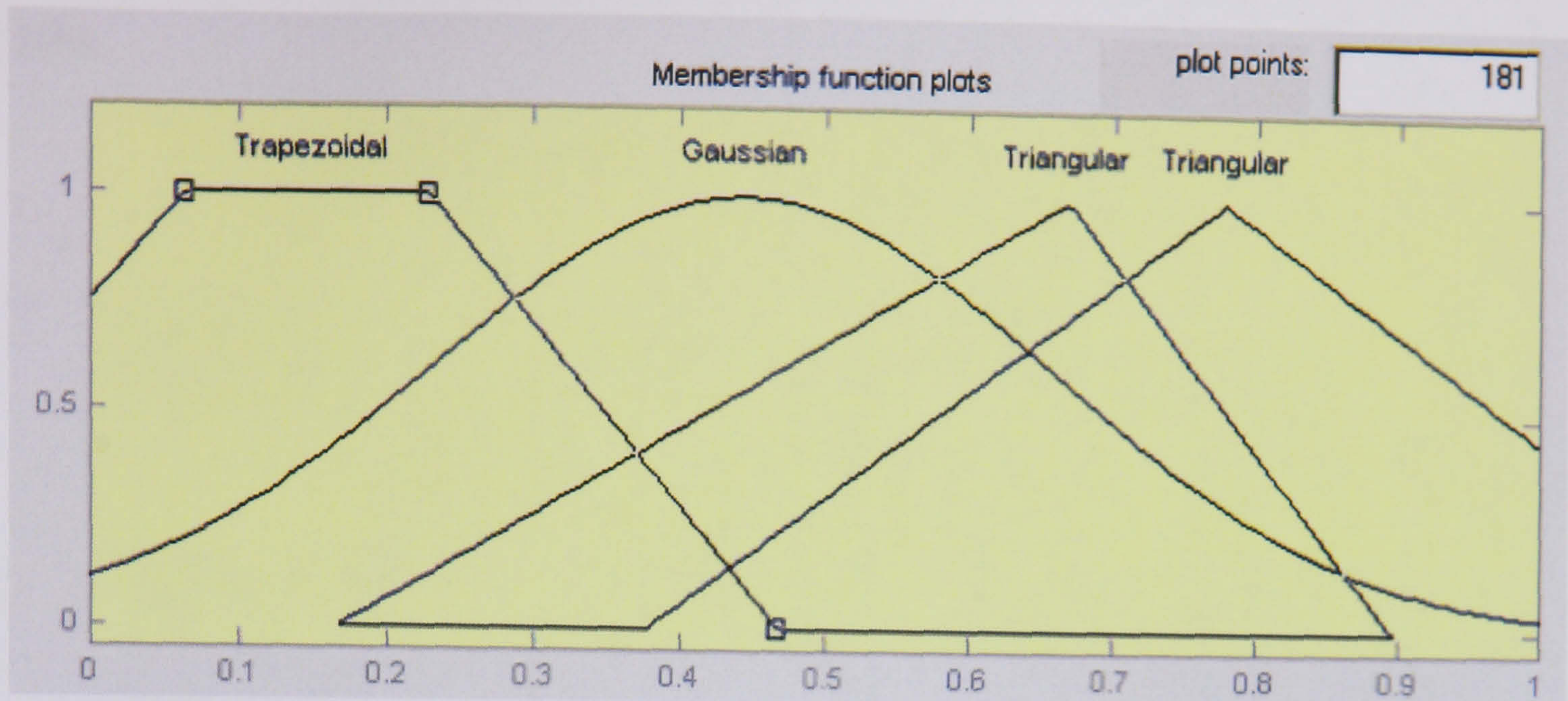
Bannatyne (1994) explains how ambiguity can then arise from the use of a vague concept such as the term “hot”, i.e. a “many values to one concept” relationship. A substantial increase in the application of fuzzy logic has occurred mainly in the Japanese commodities sector where it is used to provide control systems (Nauck et al 1997, Ross 1995 and Terano et al 1992) for such items as automatic focussing cameras and washing machines.

**Figure 3.1: Fuzzy Sets from a Screenshot in MATLAB (the Labels Commonly Stand for Low, Nearly Low, Medium, Nearly High, and High).**





**Figure 3.2: Examples of Fuzzy Set Shapes (in MATLAB).**



### 3.3 Structure of Fuzzy Logic Models

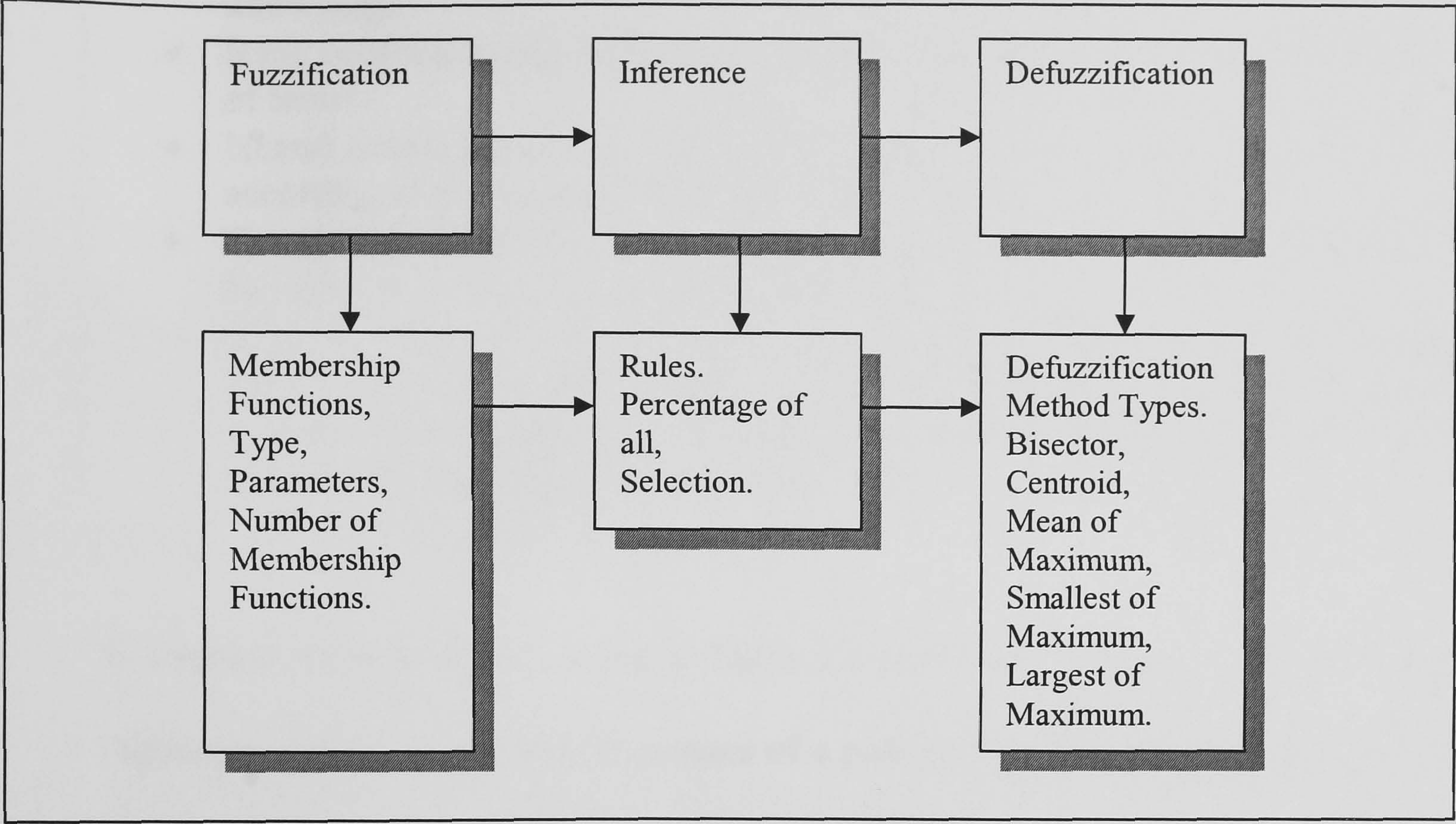
Fuzzy logic models occur via three sequential stages (Stockton and Wang 1999, Neuroth et al 2000, Roychowdhury and Pedrycz 2001) as shown in Figure 3.3 i.e. these stages are fuzzification, inference and defuzzification. Referring again to Figure 3.3, below these stages are shown concerns within these stages. These concerns form part of the experiments in Chapters 4 and 5.

Murata (1995) uses fuzzy logic to predict the actual 'Mean Time Between Failure' (MTBF) for an item of airborne electronic equipment (an Air Data Computer) and compared it to the design MTBF. The fuzzification consists of 3 fuzzy sets per input variable, of which there are also three. The rule base was developed using expert opinion, mainly through use of the "failure mode and effects analysis" technique. Defuzzification employed the centre of gravity mathematical technique. An iterative process was used to identify the most appropriate shape of the fuzzy sets in the output



variable which estimated the MTBF. The fuzzy logic system was able to estimate within  $\pm 10\%$ .

**Figure 3.3: Structure of a Fuzzy Inference System.**



### 3.3.1 Fuzzification

Fuzzification is the process by which the individual values of each input and output variable become associated to one or more, fuzzy sets (Section 3.6.2, 3.6.3, 3.6.4, and 3.6.5 provide examples of how the structure of fuzzy sets, occur in applications in the literature). The degree by which an element fits into a fuzzy set is defined by the structure of the membership functions, i.e. the number of and shape of the individual fuzzy sets. The literature identifies several methods by which membership functions can be created, including: knowledge acquisition (Watanabe 1979, Turksen 1991), Genetic Algorithms (GAs) (Oh and Pedrycz 2002), and algorithmic techniques such as clustering (Chiu 1994). A list of such methods is provided by Ross (Ross 1995), i.e. Table 3.1.



**Table 3.1: Methods for Creating Fuzzy Sets (Ross 1995).**

- Intuition using visual inspection and general reasoning, asking such questions as should the fuzzy set be symmetric, and / or does it make sense to use discontinuities within the fuzzy sets?
- Inference, i.e. by using a simple algorithm based on a known body of knowledge.
- Rank ordering using consensus or paired comparisons to provide an ordered list of terms.
- Neural networks, i.e. to evaluate the parameters of a fuzzy set and update these according to the actions of a neural network.
- Genetic algorithms, i.e. to enable the optimum values of the parameters of a fuzzy set to be chosen by genetic algorithm.
- Angular fuzzy sets in which terms are placed on the angle of a point on a unit circle. Membership values correspond to the projection of this point on the circle onto the vertical axis of the circle. The method is useful to capture the cyclic property in quantities that vary in this way.

In addition to the methods listed in Table 3.1 Leung et al (Leung et al 2003) describe “situation analysis” in which the causes of a problem are identified and used to provide the linguistic terms for each fuzzy set. In addition, Leung et al provide an example of the use of past experience to convert collected data in unsuitable formats, e.g. in diagram form, to suitable forms for developing fuzzy logic systems.

### **3.3.2 Rules**

The variety of the form of rules in literature-based applications, are provided in Section 3.6.8. The use of an “incomplete” rule set is possible. Rule reduction is an issue within Fuzzy Inference Systems when there are potentially a large number of possible combinations of fuzzy sets. The reduction in rules required to develop a rule base implies a reduction in computational expense and a reduction in the complexity of the model identification process, e.g. data collection.



Rule reduction can be achieved using

(i) A change in logic. (Combs and Andrews 1998, Mendel and Liang 1999), as shown symbolically in equation 3.3.

$$(p \wedge q \Rightarrow r) \equiv (p \Rightarrow r) \vee (q \Rightarrow r) \quad (3.1)$$

Using Equation (3.1) exponential growth becomes linear when variables and fuzzy sets are added to the model because the computation of the left hand side of Equation (3.1) is different from the computation of the right hand side. It is important to note that Equation (3.1) cannot be used to directly convert one rule base into another. The term  $(p \wedge q \Rightarrow r)$  will invoke a different part of  $r$  than just  $(p \Rightarrow r)$ .

(ii) Orthogonal transforms (Setnes and Babuska 2001) in which rule reduction is accomplished as illustrated by an example of the FIS built for a control application. The FIS was represented by a matrix equation. Subsequent examination of this equation, identified that the matrix performed the function of the rule base, (i.e. contained the firing strengths of each rule). Analysis of this matrix identified those rules that had least effect on the output and which could therefore be removed without a significant effect on estimating accuracy.

(iii) Interpolation within a sparse rule base, i.e. one in which there are a relatively low number of rules since the majority of rules are missing (Koczy and Hirota 1997, Koczy et al 1999, Tikk and Baranyi 2000, Tikk 2003). Performing interpolation between existing rules can offset the effect of the missing rules.

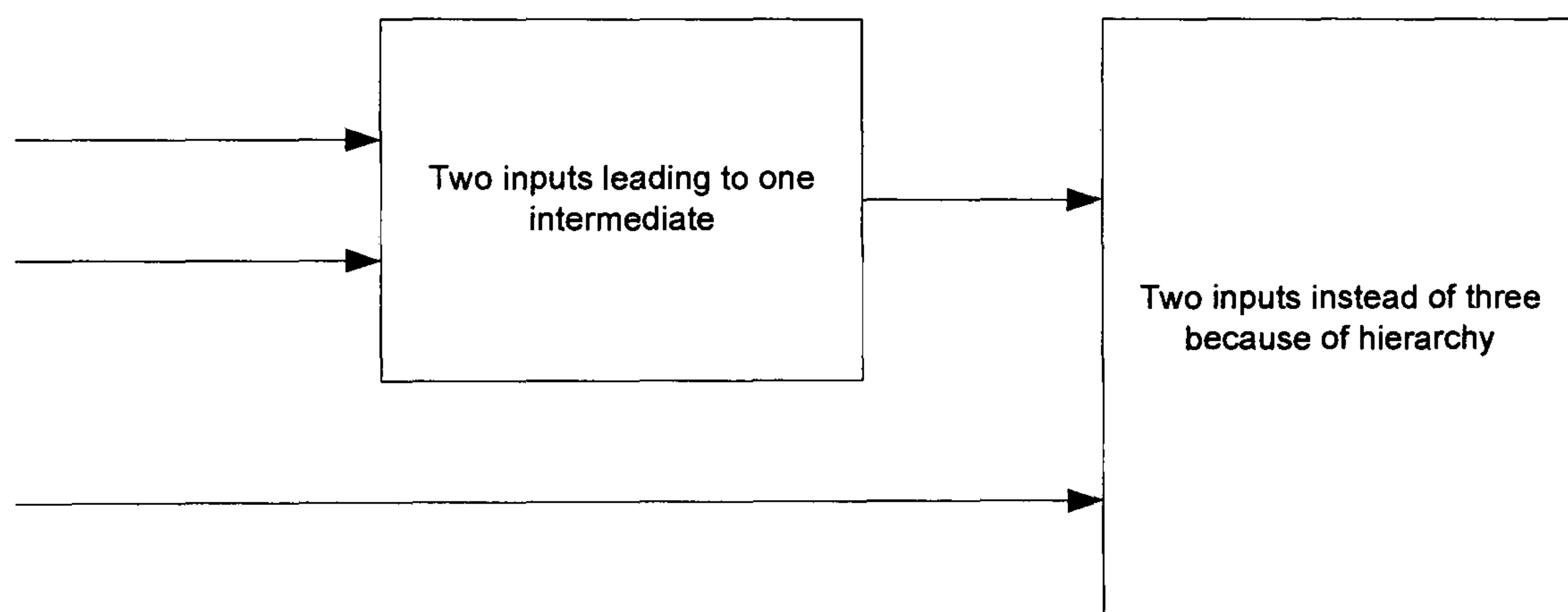
(iv) The characteristic point method, fusing and similarity measures (Setnes and Babuska 2001) all join fuzzy sets together in order to reduce their number and hence effect a reduction in the rule base.

(v) Structuring the rule base into a hierarchy (Lee et al 2003) as shown in Figure 3.4.

The hierarchical structure involves intermediate variables that can have little meaning in practice. The hierarchy means the effect of adding variables and linguistic “terms” is additive rather than multiplicative.

(vi) The use of Type-2 fuzzy sets (see Section 3.7.3).

**Figure 3.4: Rule Reduction Through Hierarchies.**



### 3.3.3 Inference

The inference process (Section 3.6.6 provides example applications in the literature) consists of two basic processes which are sequential, i.e.

- (i) implication (3.6.7 provides example applications in the literature), and
- (ii) aggregation.

The implication process is described mathematically in Appendix A3.1. Full details of the fuzzy logic inference process is found in Jang et al 1996, and other similar texts.



For a summary of mathematical operations of fuzzy logic operators, like maximum or minimum, see Appendix A1.

#### **3.3.4 Defuzzification**

The defuzzification process (Section 3.6.9 provides example applications in the literature) later chooses just one value for the output variable that is not fuzzy but is termed as a crisp result, i.e. is not fuzzy. Defuzzification is performed, as in practice only one answer is needed, (e.g. a speed response for an application involving movement, or indeed a cost estimate for a given specification)

Typically within control system fuzzy logic applications, the centroid method is used in order to provide smoother transitions between output values (Bannatyne 1994).

### **3.4 Comparison of Alternative Fuzzy Logic Methods: To Show the Variety of Forms of Fuzzy Logic.**

Examples of the many types of fuzzy logic that are used within manufacturing are listed in Table 3.2.

**Table 3.2: Types of Fuzzy Logic.**

Method	Application	Description
Fuzzy Relational Modelling Qian et al (1995)	Improving quality or reducing costs of a wood chip refining process	More than one dimensional fuzzy sets are used to represent the relation between two or more variables.
Fuzzy Ranking Jiao and Tseng (1998)	Concept evaluation in configuration design for mass customisation	Ranks design alternatives at the conceptual design stage
Possibility Theory Utkin et al (1997)	Reliability analysis of systems	Degrees of membership between 0 and 1 are given to elements of a set. The values are not constrained to sum up to 1. Applied when assumptions for the use of probabilities are not satisfied.
Fuzzy Numbers Crowe et al (2002)	Estimating risk in business process re-engineering	A triangular fuzzy number was built from subjective ratings of concepts from a questionnaire. The fuzzy number was dissected using the concept of alpha decomposition to aid in assessing a risk quantity.
Fuzzy Clustering Joisen and Liao (2000)	Part family and machine cell formation	Two types of fuzzy clustering used were “fuzzy c means” and “fuzzy k nearest neighbour” in a group technology application. The fuzzy approach was used due to the presence of non-binary data. Fuzzy k nearest neighbour was used to improve the results of fuzzy c means.
Fuzzy Control Burns (1997)	Controlling robot end effectors	Control of a robot gripper is provided using four inputs. The number of rules employed was independent of the number of fuzzy sets in the input variables.

**3.4.1 Mamdani, Takagi Sugeno Kang, ANFIS and Subtractive Clustering Methods.**

A popular subset of all the fuzzy logic methods was chosen for further consideration. The principal methods used within the area of control systems are the Mamdani method (Pham and Castellani 2002), Takagi Sugeno Kang (TSK) method, subtractive clustering



(Chiu 1994) and ANFIS (Jang et al 1997). A comparison of these methods is provided in Table 3.3.

**Table 3.3: A Direct Comparison Between the Mamdani, TSK, Subtractive Clustering and Adaptive Neuro-Fuzzy Inference System Methods.**

Method	Fuzzification	Rules	Inference	Defuzzification
Mamdani	Expert knowledge and subjective opinion.	Antecedents and consequents are fuzzy sets.	Inference can lead to the combination of fuzzy sets.	Typically the centroid method, although others can be used.
TSK	Experts, algorithms and induction.	Antecedents are fuzzy sets, consequents are polynomials.	Inference can lead to the combination of singletons.	Weighted average method.
Subtractive Clustering	Data points and an algorithm produce Gaussian fuzzy sets	Rules are built by splitting clusters up into the input and output	Inference uses the product method for implication (in this research).	Weighted average method (in this research)
ANFIS	Initial membership functions can be rough approximations that are tuned by a “neural network”	Rules are part of the “neural network” structure and hence cannot be altered	Inference use the product method for implication	Weighted average method

### 3.4.2 Mamdani Method

The structure of the Mamdani method is found in Appendix A1.2. In practice a wider choice of methods (also known here as structural elements) are available for implication, aggregation and defuzzification (see Section 3.6 for a variety of structural elements in the literature). The essential differences between the Mamdani (Cao et al

2001, Kickert and Mamdani 1978, and Mamdani and Gaines 1981) and the Takagi Sugeno Kang (TSK) methods lie in the specific use of different elements in their structure. In comparison with Mamdani, the TSK method typically uses:

- (a) the AND method is the product, and not the minimum
- (b) the OR method is the probabilistic operator, and not the maximum
- (c) polynomial functions and not fuzzy sets represent the linguistic terms of the output variable, which are typically constant or linear functions of the input variables, and
- (d) a non-centroidal defuzzification method, i.e. the weighted average method, is used

### **3.4.3 Takagi Sugeno Kang Method**

The TSK (Zeng et al 2000, Jang et al 1997 and Tanaka et al 1998) method is usually employed when historical data is used in building linear or constant output functions of the input variables (see Sections 3.4.4 and 3.4.5 where the TSK structure occurs within the ANFIS and subtractive clustering methods respectively). The input membership functions, rules and other fuzzy logic structural elements can combine in order to provide non-linear surfaces. These non-linear surfaces can result from the input of human expertise in the input fuzzy sets controlling the output of simple linear functions of these input variables or constants. Hence, the TSK method is capable of using historical data to build functions for the output variable and process subjective expertise to build membership functions for the input variables. A complex surface or function can, therefore, be generalised from the use of simple elements, e.g. linear functions and/or constants describing the output variable (Jang et al 1996).



#### **3.4.4 Adaptive Neuro-Fuzzy Inference System (ANFIS)**

The TSK FIS has been described previously in Section 3.4.3, as this is the FIS used during the ANFIS method. Neural networks are a system containing processing elements and weighted connections between them. The system learns a relationship between inputs and outputs via training data that alters parameters within the network structure. This structure is subsequently validated. A full description of neural networks is given by Jang (1997). ANFIS (Jang 1993, and Lin and Liu 2001) is considered different to a neural network, but is still a network structure, and has no weights on the connections between nodes. ANFIS is a network that maps to a FIS structure. The structure of the FIS produced by ANFIS is shown in Appendix A1. Lo (2003) used triangular membership functions in an ANFIS model to predict surface roughness from spindle speed, feed rate, and depth of cut in a milling process obtaining an accuracy of up to 96%. The ANFIS method altered initial membership functions, in one instance creating a non-normal triangular fuzzy set, i.e. with a maximum degree of membership less than one. This is an example where the meaning of fuzzy sets altered by ANFIS is a key problem in its application, i.e. there can be a trade-off between model accuracy given by ANFIS, but its lack of use by domain experts in being able to attribute meaning to its elements.

#### **3.4.5 Subtractive Clustering**

Subtractive clustering is a clustering algorithm introduced by Chiu (1994). The algorithm is chosen for this research as it produces potentially less clusters and rules than the other clustering methods shown in Table 3.2. The clustering algorithm is known for its ability to produce a low number of rules as it departs from the usual

structuring of the input space of a model into a grid (Paiva and Dourado 2001), for example this is the way of the mountain method (Ross 1995). The algorithm treats each data point in a data set as a potential cluster centre and subsequently produces clusters so that data points belong to each cluster to a degree. Park and Han (2004) explain how after clustering the data, a linear least squares optimisation process can be followed in which each cluster is treated as a Takagi Sugeno Kang model (described in Section 3.4.3) with one rule (from one cluster), and the coefficients of the linear polynomial corresponding to each TSK model output are optimised. A key parameter in the subtractive clustering method (Demerli et al 2003) is the “radius of influence”, that can be adjusted to coincide with expert opinion, of the physical influence of predictor variables on the output. This research uses the same subtractive clustering based algorithm, that first clusters the data, and subsequently produces several local TSK structures that are optimised using the linear least squares method. It is termed the “clustering method”.

### **3.5 Significance of the Shape of a Fuzzy Set**

Fuzzy set shapes can in some examples, lose connection with their linguistic terms (Mitaim and Kosko 2001, Lo 2003). It is contended in this research that accuracy of a Fuzzy Inference System is a complex interaction of all its parts, including the wider concerns of their generation, and meaning.

There are particular examples of the importance of linking the inputs to outputs to improve the required response of the model.

1. Constants are used as outputs in ANFIS for ease of training (Fuzzy Logic Toolbox Tutorial for MATLAB 6.1 from the MathWorks)



2. The advantage of using TSK is that linear functions of the input variables appearing as outputs contribute to a non-linear response because of the shape of the fuzzy sets as inputs (Fuzzy Logic Toolbox Tutorial for MATLAB 6.1 from the MathWorks).
3. The degree that an input value matches a fuzzy set contributes to the degree that the fuzzy set shape in the outputs contributes to the model's response.
4. ANFIS (Jang et al 1997) allows the shape of fuzzy sets to change by including their parameters in a "neural network" learning algorithm.
5. Chiu (Chiu 1994) links clusters that are fuzzy set shapes to accuracy of the model via a clustering algorithm.
6. John (John 2000) gives two statistical based methods for determining membership function shape, i.e. (1) frequency based and (2) direct estimation (Wattanabe 1979). Both use experts, the first may be exemplified by measuring the percentage of yes and no responses to a fixed question of whether an element belongs to a set; the second uses a direct grading of membership to a set that may be polled between experts for further iteration.

### **3.6 Fuzzy Inference System Structural Elements Used in Previous Research**

Application of FISs and the majority of applications of fuzzy logic have been in the area of control systems and these include:

- (1) the control of a steam engine (Bandemer and Gottwald 1995)
- (2) control of a cement kiln (Mamdani and Gaines 1981), but also in areas such as
- (3) facilities layout planning (Karray et al 2000)
- (4) inventory planning (Yao and Chang 2003)
- (5) design (Durr and Schramm 1997)

(6) concurrent engineering (Shehab and Abdalla 2001)

To show how previous research has used fuzzy logic the basic process of fuzzification, inference and defuzzification is examined in this section (with respect to literature based examples), with the aim of identifying the range of structural elements used.

### **3.6.1 Fuzzification**

Fuzzification occurs through (1) type of fuzzy set, e.g. triangular, (2) number of fuzzy sets, (3) parameters of the shape of the fuzzy set, (4) overlap of the fuzzy sets, (5) shoulder width, and (6) symmetry.

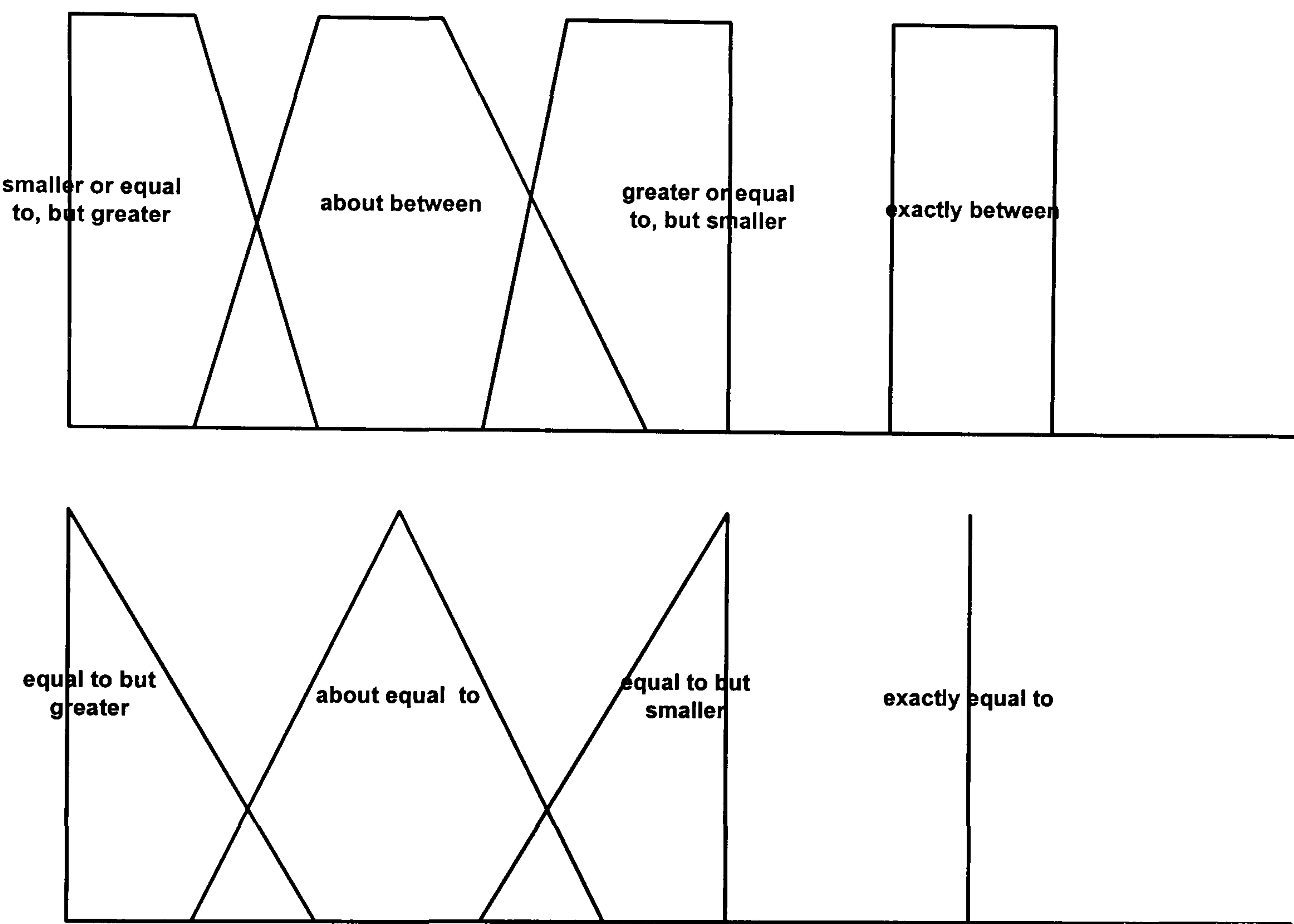
### **3.6.2 Type of Fuzzy Sets**

Harris (2001) uses triangular fuzzy sets for modelling the quality control of castings. Ming et al (1999) use triangular fuzzy sets for tool wear length estimation. These sets are optimised by a Genetic Algorithm. Li et al (2002) use fuzzy sets to model gradual transitions between strategic factors in a marketing application. Trapezoidal sets are used for simplicity. Liao (1996) used trapezoidal membership functions to model approximate ranges of values, and triangular membership functions to model just approximate values. The application is in material selection via multi-criteria optimization. Liao's (1996) fuzzy sets can be seen in Figure 3.5. The use for comparative purposes explains their linguistic labels. Jahan-Shahi et al (1999) use fuzzy logic for time estimation in flat plate processing. Recalibration is suggested of a time estimating model to a particular factory by changing the shape of the fuzzy sets (e.g. labour skill level), but keeping the logic or rules the same. Dimitrovski and Matos



(2000) use trapezoidal fuzzy sets in economic analysis of capital outlay for equipment, as they involve ranges of values occurring between truth-values of 0 and 1 (interpreted as possible variation) and 1 itself (interpreted as most likely variation).

**Figure 3.5: Liao’s (1996) Fuzzy Sets.**



Such a choice was influenced by the use of interval mathematics in previous applications. Jahan-Shahi et al (2001) further discuss their use of multi-valued fuzzy sets to represent uncertain process activity time variables for Flat Plate Processing (FPP), e.g. skill. They argue against assigning just one truth-value per value of the range of a fuzzy set as this loses information. Instead they have one fuzzy set called the Average, one called Concentration Contrast Relaxation (CCR) and one called Dilution Contrast Intensification (DCI). CCR can be achieved by squaring truth-values of the

Average fuzzy set and DCI by square root of the truth-values of the Average fuzzy set among others, for example. The result is a fuzzy set that has a range of truth-values instead of just 1 value, for each point of the range of the fuzzy set. The Average fuzzy set gives a point truth-value somewhere in the middle of the range given by CCR and DCI. Neuroth et al (2000) use fuzzy logic to control the pumping of oil. Examples are given of fuzzy sets that are triangular and low in number. They are symmetrical and are placed so that the apex of a fuzzy set is midway between the apexes of the neighbouring fuzzy sets. The endpoints of the fuzzy sets are placed so that they coincide with the apex of neighbouring fuzzy sets. The fuzzy sets are therefore placed uniformly. Xu et al (2002) use fuzzy logic for colour classification. The fuzzy sets in the input variables,  $R_d$  (brightness) and  $b$  (yellowness), were Gaussian. One more fuzzy set was used for  $b$  than  $R_d$ , as it was considered more important for classification. No reason was given for the use of the normal distribution for membership functions, though it was suggested that their parameters could be altered by previous data sets of input and output vectors. Overlap was increased between neighbouring membership functions to indicate how the boundary between two linguistic variables was blurred. The output fuzzy sets were triangular to make defuzzification simpler. In addition they were symmetrical and placed exactly side by side with no overlap because their function was to perform a classification that was to be “clear and unbiased”. Cox (1994) uses fuzzy logic to model price. The fuzzy sets used are simple and low in number. Reasoning, for example, is used to decide on the width of a fuzzy set representing, “competition price”. The smaller the width the more a required price should correspond to the competition price.

**Summary of Types:** triangular, trapezoidal, square, singleton (single valued), interval valued, symmetrical, uniformly placed, Gaussian.



### 3.6.3 Number of Fuzzy Sets

Kodagoda et al (2002) use eleven triangular fuzzy sets (a high number compared to most applications) within each input variable and six in the output variable for their fuzzy driver controller module of an Automatic Guided Vehicle. Triangles are said to be simple and easy to optimize. Chan et al (1997) use simple linear “non-increasing” and “non-decreasing”, i.e. 2 fuzzy sets per estimating parameter in a linear programming method, when building a fuzzy regression analysis model for estimating waste treatment costs. Lau et al (2002) use a combination of fuzzy logic and neural networks in order to calculate the numbers of units required from supply chain companies. They use the concepts of quality and quantity. The fuzzy sets are built using “field knowledge and past experience”. The fuzzy sets shown for supply change rate are triangular and number seven in total. Fei and Jawahir (1993) use five fuzzy sets to model chip breakability and were assigned as linguistic terms. Chip control diagrams were represented within a two dimensional space of depth of cut versus feed-rate. Because of the gradual change in chip breakability then linear interpolation could be used to assign chip breakability linguistic terms to places within the chip control diagrams. Clustering is facilitated by assigning membership degrees to fuzzy sets based on distances between points in the two dimensional space. Hui et al (2002) use fuzzy logic for the purposes of line balancing in the apparel industry. Only two input variables or concepts are used, namely, “difference in buffer level” and “difference in section performance”. The output variable was, “number of operators to move between sections”. The fuzzy logic structural elements were chosen by interviewing a number of industrial experts. Triangular fuzzy sets were used with experts naming nominal values that referred to the linguistic label of the fuzzy set. The fuzzy sets overlapped uniformly. The uniform

overlap was confirmed as the fuzzy sets chosen by the process experts by interrogating them using language that was not related to fuzzy sets. Trapezoidal fuzzy sets were not used as the experts could not agree on the range that would correspond to the linguistic variable having a truth value of one.

**Summary of Numbers of Fuzzy Sets per Variable:** 11, 6, 2, 7, 5,

### **3.6.4 Parameters of Fuzzy Sets**

A parameter of a fuzzy set is a variable that helps to describe a fuzzy set algebraically. Examples might be the equations of the two straight lines that describe a triangular fuzzy set, or the locations of the base and apex of a triangle. Klir and Folger (1988) use an algorithm that is built subjectively when modelling precipitation levels. Khouja and Booth (1995) use an objective function to estimate the degrees of membership of a robot to a fuzzy set. The membership value of each robot in a cluster was constrained to add up to one and was always to be larger than zero. Khoo et al (1995) use intuition to help in building fuzzy sets for a complex electroless nickel plating process. Liao (1996) used a pair wise comparison method to determine linguistic variables and placement of fuzzy sets in an application of material selection. Scwarc et al (1997) use a simple linear relationship involving costs of material handling and machine processing. The relationship is applicable because of assumptions made in building a linear programming model, i.e. machine capacity is assumed to be lower than available. Half trapezoids for fuzzy sets are constructed to represent a fuzzy objective function. Ming et al (1997) used subjective judgement, trial and error and subsequently a genetic algorithm to tune in the parameters of fuzzy sets and rules for tool wear length estimation. Jahan-Shahi et al (1999) use a histogram of times to decide on the range of



their output variable in flat plate processing. Dimitrovski and Matos (2000) use alpha cuts in accordance to the model users' choice of uncertainty, in order to produce outputs from the model in engineering economic analysis. Kahraman et al (2000) use triangular fuzzy sets to model vague statements about quantities in a cost benefit ratio determination, e.g. interest rates. Such vague statements included: "approximately between 10% and 15%". The fuzzy sets are used instead of statistical methods that, in contrast, use crisp figures such as 12.5%. The triangular fuzzy sets are handled by splitting them into a left hand side and right hand side. Collantes et al (2000) use fuzzy logic to analyse quality in steel making. The use of process experts and statistical methods are cited as methods of building fuzzy sets; and advanced methods such as neural networks are stated as methods for building rules. The latter are criticized for their lack of use of subjective opinion and hence difficulty in their understanding. Ntuen (1999) uses fuzzy logic to make predictions about cognitive tasks within the electronics industry. Both experts and novices were observed. Two approaches were used to build fuzzy sets. One was based on a statistical method the other was based on some subjective opinion. Lau et al (2002) use fuzzy logic for the prediction of units needed from a supply chain. The fuzzy sets are built using "field knowledge and past experience". The fuzzy sets shown for supply change rate are triangular and number seven in total.

**Summary of Methods to Choose Parameters of Fuzzy Sets:** algorithm, objective function, constraint in adding up degrees of uncertainty, intuition, pair-wise comparison, subjective judgement, trial and error, Genetic Algorithm, histogram of empirical data, alpha cuts, neural networks, and statistical methods

### **3.6.5 Overlap, Shoulder Width and Symmetry of Fuzzy Sets**

Fuzzy sets can possess symmetry, or overlap with each other (or not) and can possess shoulder width, for example the shoulder of a trapezoid is its plateau. Harris (2001), for the quality control of castings, make overlaps between fuzzy sets proportional to a parameter, namely wall thickness. Li et al (2002) begin by the use of equally spaced, equally overlapping, symmetrical trapezoidal fuzzy sets for the concepts of “business strength” and “market attractiveness” when developing a marketing strategy. Interaction with the model is encouraged in the form of the users modifying the membership functions. Ming et al (1997) initially partitioned the input space equally before training with a Genetic Algorithm when estimating tool wear length.

**Summary of Aspects of Choice of Overlap, Shoulder Width and Symmetry of Fuzzy Sets:** proportionality to variables, equally spaced, equally symmetric, equally overlapping, adaptation through Genetic Algorithm, subjective judgement.

### **3.6.6 Inference**

Inference occurs through (1) implication, and (2) rules, including their number and how they are formed. After implication, the modified output fuzzy sets for the rules that have fired, are aggregated together.

### **3.6.7 Implication**

Ming et al (1999) used the product operator when estimating tool wear length. Kodagoda et al (2002) use a Mamdani inference method for a control application. Jahan-Shahi et al (1999) use Mamdani inference for time estimation in flat plate processing and do not indicate why. Neuroth et al (2000) use implication, aggregation



and defuzzification methods that are all typically Mamdani. The application is a control application, i.e. of pumping oil, and therefore explains such a choice. The control using fuzzy logic is smoother than that for the conventional system. Importantly the fuzzy logic controller only comes into effect at certain times to produce this smoothness. In this way the fuzzy logic improves and works with the conventional controller. Xu et al (2002) in, their colour classification for cotton, make a choice of aggregation operator. The aggregation operator used was the OR operator in order to ensure that the highest membership value was taken when combining more than one fuzzy set in the output. Hui et al (2002) are concerned with line balancing. The implication and aggregation of the Mamdani method was chosen due to its popularity in the literature. The structure of the Mamdani method can be found in Appendix A1.

**Summary of Aspects of Implication:** product, Mamdani inference.

### **3.6.8 Rules**

Ming et al (1997) used a Genetic Algorithm to tune in rules in estimating tool wear length. An individual within the GA process is a rule represented by a series of centre points and widths of the input triangular membership functions and the output of the rule is a single value. Collantes et al (2000) use fuzzy logic as an expert system for analyzing quality within steelmaking, more particularly the secondary steelmaking process. The case in question was the improvement of an existing fuzzy model. The improvement involved the addition of new input variables, hence linguistic variables and hence more rules. The possible number of rules increased to 1475. This larger number of rules was circumvented by using experts to delete “infeasible” rules while trying to retain “realism”. The software “fuzzy tech” was used to check for

completeness after this process without saying how exactly this was done and what was the criteria. Therefore process expertise was used to reduce the rather large rule base. Both the rules and the fuzzy sets were tuned using data from the system being modelled using process experts. Hadie (1993) does not use rules. Fuzzy sets are given that model the goals and constraints of extending an assembly line. Using a function that maps the space in which the constraints are defined into the space where the goals are defined, the fuzzy sets for both constraints and goals are combined using the intersection operator to arrive at a decision set. The value with the maximum membership is then taken from the resulting decision set to find the optimum decision. Xu et al (2002) use fuzzy logic for colour classification in cotton. The number of fuzzy sets was kept down in the input variables so as to limit the number of rules. The rules were established by the use of experience but also using colorimeter data. The connective AND was used in the rules as both input variables were being considered at the same time. Lau et al (2002) use fuzzy logic for procurement. It was noted that all combinations of the fuzzy sets in two input concepts are used in forming the rule base. Different fuzzy sets from supply change rate are stated for the output variable. Here the rules are created on the basis of experience from field experts, experimental results and theoretical deviation. It is noted that the number of combinations of the input fuzzy sets in the input variables is low to form a complete rule base. Cox (1994) uses fuzzy logic for pricing. Each unconditional rule was worked through producing an output fuzzy set, “price”, for each one. All the unconditional rules were aggregated using the AND operator, i.e. the output fuzzy set, price, was gradually built up. Conditional rules were worked through to arrive at a consequent set that was combined with the output fuzzy set price, arrived at earlier, via the union or OR operator. Khoo et al (1995) created rules for a complex system from



interviews with operators, a literature review, and empirical results. In total only 294 heuristics were included with three parameters to control a complex nickel electroplating process and all its relationships. Liao (1996) did not use any rules or inference when using operators on fuzzy numbers for ranking materials for selection. Ming et al (1997) use 44 rules to estimate tool wear length. In Kodagoda et al's (2002) control applications there is a complete rule base in that each and every combination of fuzzy sets in the input variables is defined by a corresponding fuzzy set in the output variable. The rule base is given in a matrix format. The complete rule base numbers  $11 \times 11$ , i.e. 121. The complexity of the overall implementation is reduced by using a number of separate controllers rather than just one. Coupling them together is achieved by using explicit coupling rules. Jahan-Shahi et al (1999) use  $3^3$  (or 27) rules but have 9 fuzzy sets to choose from in the output variable in their flat plate processing problem. Neuroth et al (2000) discuss rules in their model for pumping oil. The rules can be considered to be low in number due to the low number of fuzzy sets and input variables. El Baradie (1995) use fuzzy logic to predict cutting speed for a metal removal process. The input and output variables were connected by only five rules that were said to imitate an experienced machine operator. Fei and Jawahir (1993) implement fuzzy logic by the production of fuzzy sets for chip breakability that were assigned to the space of depth of cut versus feed rate based on an algorithm. The algorithm assigned degrees of membership to at most two of the fuzzy sets based on distances within the space. The result was a rule set that was not explicitly given but appeared in a two-dimensional space and could almost be read like a Fuzzy Associative Memory (FAM) (a FAM is a rule-base in the form of a "n-dimensional" matrix that relates inputs together, and whose entries are outputs). Hui et al (2002) use fuzzy logic for line balancing. The rule base

was complete and low in number, i.e. twenty five. A long time was taken using process experts to build the rules since they were considered crucial for the success of the model. It is noted that the model has a low number of rules and no indication has been given as to how a bigger and more complicated model may be built. Cox (1994) uses fuzzy logic to model price. A low number of rules were given by experts that numbered less than ten. One rule did not necessarily involve all the fuzzy sets. Rules were categorised as conditional or unconditional.

**Summary of Aspects of Rules:** tuning via Genetic Algorithm, “curse of dimensionality”, choice by subjective judgement, data driven choice, connectives (e.g. AND), conditional and unconditional rule types, 294 rules used, 44 rules used, complete and incomplete rule bases, 121 rules used, variable reduction means rule base reduction, 5 rules used, 25 rules used, 10 rules used.

### **3.6.9 Defuzzification**

Defuzzification occurs through the following methods, (1) bisector or centre of area method, (2) centroid or centre of gravity method, (3) Mean of Maximum, (4) Largest of Maximum, (5) Smallest of Maximum, (6) weighted average, and (7) many others.

Jang et al (1997) state the five most popular defuzzification methods: bisector of area, centroid of area, mean of maximum, smallest of maximum, and largest of maximum.

Hui et al (2002) uses fuzzy logic to model line balancing. The centre of area method (also known as the bisector of area) was chosen as defuzzification but not justified.

Harris (2001) used the centroid method for defuzzification in a quality control application. The centroid method resembles a fuzzy version of probabilistic expectation (Jang 1997) that explains its popularity (the resemblance is in weighting variable values



with a proportion of the uncertainty). But it cannot be over emphasised that fuzzy logic and probability are distinctly different concepts. Khoo et al (1995) used the maximum value in terms of certainty for defuzzification in their complex electroless nickel plating process. Xu et al (2002) make a choice of defuzzification operator for their cotton colour classifier. The defuzzification method used was the centroid as this was most popular. Cox (1994) use fuzzy logic for pricing. The centroid method was chosen as a defuzzification method since the solution moved around more smoothly in the output space. Using a strategy that involved the maximum of fuzzy sets was, in contrast, said to move around more erratically and hence was less smooth. Kodagoda et al (2002) use the centre of gravity method for control of an AGV. Kahraman et al (2000) use a number of ranking methods to convert triangular fuzzy sets into a single quantity. The application is in the determination of benefit to cost ratios. The parameters of a triangular fuzzy set are combined, for example parameters a, b and c are combined by Chiu and Park's method (Chiu and Park 1994) to make the single quantity:

$$((a + b + c)/3) + wb,$$

where:

a is the smallest value over the range of the fuzzy set,

b is the value of the fuzzy set that has certainty of 1,

c is the largest value over the range of the fuzzy set, and

w is a value, "determined by the nature and the magnitude of the most promising value".

Hadie (1993) uses fuzzy logic for economic based decision making concerning the addition of machines to assembly lines. Defuzzification occurs by choosing the alternative from the decision set with the highest membership value.

**Summary of Defuzzification Types:** bisector, centroid, Mean of Maximum, Smallest of Maximum, Largest of Maximum, Chiu and Park's method (Chiu and Park 1994), maximum for decision making.

### **3.7 Influence of Fuzzy Logic on Cost Model Characteristics**

The basic characteristics of a cost model have been previously discussed in Section 2.3. This section now examines the potential influence of the use of fuzzy logic on these characteristics in terms of benefits and advantages or disadvantages. The cost model characteristics are particularly examined under the different headings of, estimating accuracy, user personnel, set-up and operating costs, and data requirements.

#### **3.7.1 Estimating Accuracy**

Fuzzy logic is a universal function approximator and hence can approximate any cost function to an arbitrary degree of accuracy. Fuzzy inference systems are universal function approximators for a variety of model structures (Mendel 2001, Koczy and Hirota 1997). For example a "singleton Fuzzy Logic System" using the product operator in rules and implication, Gaussian membership functions, and height defuzzification. Universal function approximation was also proved in the literature by Kosko (Kosko 1994) for "additive fuzzy systems", i.e., singleton fuzzification, the product operator for rules and implication, the concept of patches, and centroid defuzzification. Other research includes Zeng et al (2000) for TSK fuzzy systems, and Wang and Mendel (1992) show fuzzy basis functions can achieve universal function approximation. It is apparent that a proof must be found for each model structure. Until recently it was thought that some FIS structures may not be suitable in attempting to achieve the



universal function approximation property. The work of H. T. Nguyen (Tokyo Institute of Technology) and V. Kreinovich (University of Texas) promises to extend the result to all FISs. It can be noted that in choosing the correct fuzzy logic structural elements, leads to an accurate approximation of a cost estimating relationship due to this universal function approximation property. But a limit is introduced by Tikk et al (Tikk et al 2001) by proving that some FISs fall short of universal approximation when the number of fuzzy sets or rules is bounded. Appendix A1 summarises Universal Function Approximation results.

Research has exemplified fuzzy logic as a possible cost modelling tool but not found its accuracy. Shehab and Abdalla (2002) provide a mechanism for the use of fuzzy logic in cost estimating. The process of cost estimating was carried out in the context of a software package developed for concurrent engineering. It has only two cost drivers available and hence combats the curse of dimensionality in this way. No indication of accuracy is given.

### **3.7.2 User Personnel**

The main tasks envisaged to be performed by users are:

- (a) collect data,
- (b) select fuzzy logic method,
- (c) set up fuzzy logic method,
- (d) select fuzzy logic structural elements,
- (e) set up rule base,
- (f) set up fuzzy sets,

- (g) justify outputs,
- (h) produce learning mechanism through iteration, and
- (i) explain relationships.

The main personnel issues are (1) Who would do it? (2) What skills / experience is required? And (3) what is the input of the cost engineer in each area?

An understanding of fuzzy logic may be difficult to impart to the process expert whom is being used in order to collect the data. Experience in industry from building fuzzy logic models using experts in drilling and plastics processing has found a first time exposure to the method could cause some confusion. The main difficulty in building a fuzzy logic model can be argued to be the building of membership functions. Obtaining values for their parameters or indeed justifying their shape is considered to be difficult. Triangular membership functions are popular, as well as trapezoidal and Gaussian functions. Simplicity and ease of understanding of the function shape and its implications for uncertainty are very important when choosing membership functions. Issues such as computational simplicity are overcome by the use of simple functions with few parameters. Choosing the right membership functions, as well as the right combination of them, is aided by a number of iterations with a process expert. Alternatively this can be done by the use of learning algorithms that might use a “neural network” and pairs of input and output data. Therefore exemplar methods of building membership functions are: built by many visits to a process expert, a more mathematical approach such as a “neural network”, or alternatively a Genetic Algorithm in order to ‘tune’ the expert knowledge.



### 3.7.3 Set-up and Operating Costs

The main setup and operating costs of fuzzy logic are envisaged to be through:

1. software
2. data collection

The method requires specialist software and knowledge. Specialist software can preclude the complete understanding of algorithms and only appreciation of certain parameters or properties. This is a particular advantage of the application of fuzzy logic through specialist software that saves time and cost. Very deep understanding of the fuzzy logic is not therefore necessary and an advantage is clearly gained. Fuzzy logic may be used via other mathematical software or even a spreadsheet but in effect it must be programmed.

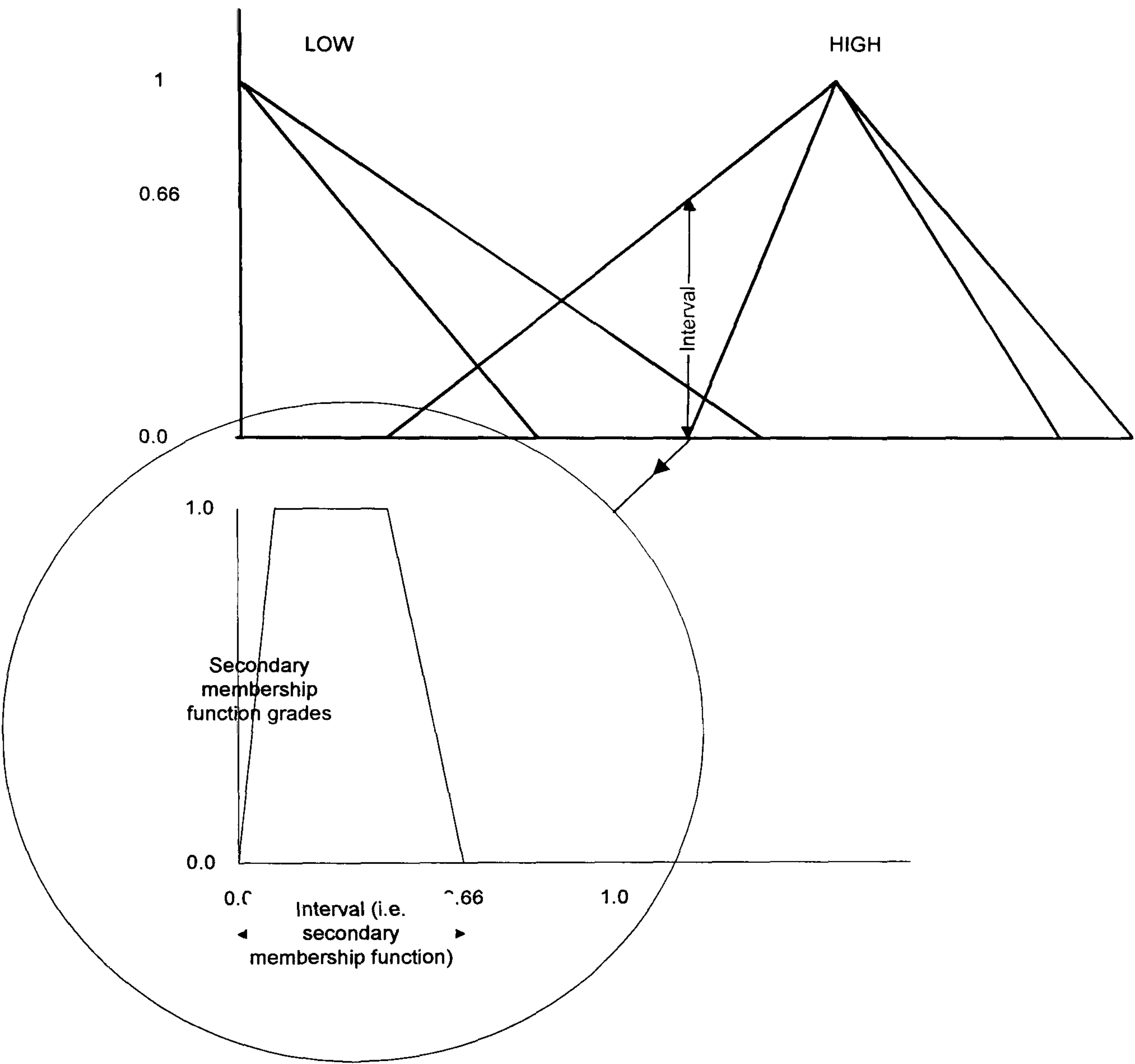
Fuzzy logic may be understood on several levels, the most notable being (1) linguistic or (2) mathematical. Once an initial model has been built it can be modified at either of the linguistic or mathematical levels. Therefore the model is not a black box technique but has high transparency. The model can function despite it being incomplete (e.g. the rule base may have certain rules missing yet the model can still give a cost). The model can be used on a linguistic or word level in order to make decisions while complex mathematics is avoided. In this way the model can be audited with greater ease than other data analysis methods.

Fuzzy logic has been used in many hybrid applications. This means that fuzzy logic has been used in conjunction with other data analysis methods (e.g. neural networks, GAs or

regression analysis) that may improve the performance of the method while still keeping the "computing with words" paradigm. This notion of fuzzy logic being used in conjunction with other methods is very important for method selection, usability and understanding of models. Of equal importance is the notion of fuzzy logic being able to model a problem at different levels. Type-2 fuzzy logic is a fuzzy logic that models the uncertainty of the uncertainty (Mendel and John 2002, Mendel 2001) but is not considered here. In particular the degree of membership of a fuzzy set is itself a fuzzy set, as shown in Figure 3.6. It is not as well documented as type-1 fuzzy logic (i.e. degrees of membership are numbers) and it involves data collection on two tiers, i.e. the uncertainty of the uncertainty. Importantly, though, type-2 fuzzy logic can be used in rule reduction, because of the ability of type-2 fuzzy sets to contain more information about uncertainty and hence effectively reduce the need for a greater number of type-1 fuzzy sets.



**Figure 3.6: Example of a Type-2 Fuzzy Set in which all Degrees of Membership are Themselves a Fuzzy Set.**



Some parameters that influence the cost of a product, may be difficult or even preclusive to measure. Others may cause very expensive data to be collected. Fuzzy logic can allow this to be bypassed and provide a structure or framework in which to consider expert opinion on these parameters. Therefore a different type of data is being collected in order to produce a less expensive cost estimate. The question is, how much is the “tolerance for imprecision” (Zadeh 1996).

### 3.7.4 Data requirements

Use of fuzzy logic causes data requirements to change from the two areas of:

1. concept stage, and
2. detailed stage.

Fuzzy logic can be expected to function in the range of areas where different levels of data detail are required.

Fuzzy logic is known for the use of qualitative data or subjective opinion. As shown in the examples in Section 3.4, fuzzy logic can be considered as a data analysis method that can utilise process expertise as a data source. More importantly still, fuzzy logic can use a mixture of data sources in order to construct its structural elements. By using the high level interpretation of the model a process expert may consider a data driven model and make changes that reflect an understanding of the process as well as using a data set. Therefore fuzzy logic can be analysed on a linguistic and mathematical level. Data driven models can be interpreted linguistically, though care must be taken (e.g. sometimes the data driven approach produces fuzzy sets with dubitable meaning (Mitaim and Kosko 2001, Lo 2003). Expert knowledge driven models can be modified by using a data set and therefore altered mathematically. The alteration may also be seen linguistically. In this way, then the two levels are being used for interpretation and data analysis. It is emphasised that further research is required to effectively understand the transition between expert driven models and data driven models.

### **3.8 The Cost Model Characteristics in Terms of Previous Research into Using Fuzzy Logic**

The cost model characteristics from chapter 2 are summarised under: (1) accuracy, (2) data collection effort, (3) subjective judgement, (4) development (cost and time), (5)



tasks (manufacturing volumes, variety of tasks, and repetitiveness of tasks), (6) detail of input data, (10) estimate application time, (11) personnel whom operate the system, and (12) cost model level.

### **3.8.1 Accuracy**

Ming et al (1997) estimated tool wear length with a mean error of less than 13%, with 2 input variables and 7 fuzzy sets each. Forty-four rules were used, i.e. less than  $7^2$  or 49, because of the use of a GA for optimisation. Kodagoda et al (Kodagoda et al 2002) showed that in a control application with 121 rules that fuzzy logic, more particularly TSK, outperformed a proportional integral derivative controller. Areas of improved performance included tracking accuracy, steady state error, control chatter, and robustness. These areas, in particular tracking accuracy and robustness, can be analogous to areas within cost estimating. Tracking accuracy corresponds to the accuracy in keeping to the correct costing curve in space, and robustness keeps to the model's ability to cope with incomplete or missing information. Dimitrovski and Matos (2000) use alpha cuts of trapezoidal fuzzy numbers to allow a measure of error in engineering economic analysis. Pedrycz et al (1999) use a Fuzzy C Means (FCM) clustering algorithm to build fuzzy sets to aid in cost modelling for software projects. The data being modelled was the “Yourdon project software data” that was highly scattered and non-linear when plotted. The FCM allowed, therefore, great improvement. It also introduced the concept of granularity of information and the visualisation of uncertainty, both via fuzzy sets.

### 3.8.2 Data Collection Effort

Data collection in the cost model development process is exemplified through literature based applications in Section 2.5. The variables in the research of Ming et al (1997), (e.g. tool wear) were determined by analysing groups of experimental data, using such quantities as dispersion and power spectral density of feed directional acceleration. A triaxial accelerometer and in turn a video tape is used to collect data from the latter. Jahan-Shahi et al (1999) time an activity type, namely plate carrying and loading in Flat Plate Processing (FPP), and find it is distributed because of the uncertainty involved in the process, i.e. different times are recorded for the same activity. The range of the variables over which they are modelled by fuzzy sets are found through experience and measurement. Other activities within FPP are considered as independent variables and so are individually modeled using other FISs. Pedrycz et al (1999) use a triangular fuzzy set as a granule of information to be input into a Constructive Cost Model (COCOMO) software engineering model relating effort (in man months) to size of a project (in Thousands of Lines of Code, TLC). The minimum and maximal size of a system is collected in order to help construct the triangular fuzzy sets used. Using the COCOMO model equation, i.e.

$$E = aS^b \tag{3.4}$$

where:

S is software size in thousand lines of code,

E is the effort in man hours for producing the project and the parameters,



a and b, take on values that depend on the classification of the project into three different types, namely embedded, organic and semi-detached systems.

the input variables and the extension principle make the output “granules” turn out to be non-linear. Quin et al (2001) use fuzzy logic at the conceptual design stage that is said to incorporate a degree of uncertainty. A sketch is said to represent vague and imprecise ideas. Xu et al (2002) use a FIS for colour classifying in cotton. The fuzzy inference system was built making use of human knowledge as well as empirical data. It was said to increase consistency within its classification.

### **3.8.3 Subjective Judgement**

Ming et al (1997) noted that validation of a model built subjectively can occur through automatically building the same model from data and an algorithm for comparison. Jahan-Shahi et al (1999) use fuzzy sets and probability to model uncertainty in time estimating, for example, labour skill level, plate carrying time, plate size and plate thickness for fuzzy logic and labour fatigue, shortage of resources and materials, failure of machines and unavailability of facilities for random variables. Fei and Jawahir (1993) introduce the problems of prediction in production planning with the advantage of knowledge about machining processes. The aim is to be able to plan with the advantage of models of the machining processes. The area of chip breakability was examined and a fuzzy classification method used to make predictions. It was assumed that chip breakability, although sensitive to small changes in cutting conditions, would also change gradually with them. Hadie (1993) uses fuzzy logic for decision making. The process of decision-making is mathematically modelled and is made more transparent

by the use of fuzzy sets. Xu et al (2002) were concerned with the colour of cotton. Fuzzy logic is used to classify the colour of cotton based on values produced by a colorimeter. The colorimeter gives values that can be plotted in a two dimensional space containing the input variables, brightness and yellowness. Previously it was found that it was difficult to classify the colours of cotton based on colorimeter readings because the boundary used was not realistic. It did not reflect natural boundaries between different colours of cotton. The problem was how to model the ambiguity in a particular colour that was around this boundary, hence the adoption of fuzzy logic and the building of a fuzzy inference system. Lau et al (Lau et al 2002) use a combination of fuzzy logic and neural networks in order to calculate the numbers of units required from supply chain companies. They use the subjective concepts of quality and quantity.

#### **3.8.4 Development**

Ming et al (1997) used 8 categories of cutting conditions for generating data for rule generation and verification of estimates. Empirical results identified variables. Hui et al (2002) uses fuzzy logic for line balancing. The fuzzy inference system built outperformed the supervisors in that production targets were met earlier and the line was more stable. The system was said to be more efficient and consistent.

#### **3.8.5 Tasks**

Jahan-Shahi et al (1999) use fuzzy sets to model the variability in times measured for a handling activity in flat plate processing. The variability was explained by subjective random factors. The activities were part of an Activity Based Costing framework.



### 3.8.6 Detail of Input Data

Chan et al (1997) use an improved fuzzy regression model for the human input into estimation of costs of wastewater treatment. Fuzziness was captured in the parameters of the regression subject to minimum vagueness by a linear programming method. Jahan-Shahi et al (1999) use both fuzzy sets and probability distributions to represent uncertainty in input variables that otherwise could not be modelled, e.g. the actions and behaviour of people. Pedrycz et al (1999) state that non-numeric measurements and estimates in software engineering coupled with the ubiquitous imprecision, made for an application in granular computing where a granule was a fuzzy set. Neuroth et al (Neuroth et al 2000) describe the application of fuzzy logic and artificial neural networks within the oil and gas industry. The ability of fuzzy logic to simplify situations where large amounts of data need to be dealt with was cited. A situation was described for the control of pumping oil along a pipe leading to transient large correction control actions occurring. The two control actions are discharge and suction. Fuzzy logic is used to model operators' actions linguistically and in such a way, so that it can be used in a large number of situations where different values occur for suction or discharge. The fuzzy logic model produces a rate of change of control action. El Baradie (El Baradie 1995) uses fuzzy logic to simplify the vast amounts of data within the machining data handbook. Because of the complexity of a metal removal process, experimental machining data provided in the handbook, as well as other sources within industry, only provide initial starting points. Fei and Jawahir (1993) use fuzzy logic in production planning. The benefits that the fuzzy model gave arose from its predictive capability and the gradual changes between chip breakability linguistic values, a feature of fuzzy modelling. The fuzzy algorithm was based on assumptions made by the model

makers in assigning combinations of distances within the space as degrees of membership to chip breakability fuzzy sets. Hadie (Hadie 1993) uses the concept of fuzzy sets to model uncertainty within decision making in extending an assembly line. It was considered how the addition of an assembly machine into a production line would influence the economics of a slightly altered product by adding another component to it. An example is given in which uncertainty in the line giving defective parts is modelled using given probabilities for the occurrence of a given event. Cox (1994) uses fuzzy logic for pricing. A low number of sometimes conflicting rules and information that was generated using experts and subjective notions only, was used to price a product. It can be noticed that the rules were not a systematic complete set as seen in other applications.

### **3.8.7 Estimate Application Time**

Mamdani (1993) stated that from reading and understanding Zadeh's paper on fuzzy sets and successfully building a fuzzy controller for a steam engine, took about a week. Such is the ease of implementation of fuzzy control. Disadvantage is given to systems that involve control but also human behavioural input, for example economic systems (inflation, unemployment) and telecommunication systems (service quality) when using analytical techniques that involve differential equations for instance. These disadvantages can be overcome and simplified tremendously using if then rules and fuzzy sets. It is expected, therefore, that estimate application time be decreased because of these advantages and associated advantages in understanding.



### **3.8.8 Personnel Whom Operate the System**

This research uses the fuzzy logic toolbox that is part of MATLAB from the MathWorks Company. Other software is available, for example Swarc et al (1997) used the Hyperlindo software package. Personnel whom use fuzzy logic will have their task greatly simplified and be protected from programming using such software as the fuzzy logic toolbox. But personnel have to be aware of the subtleties that can occur when using fuzzy logic. For example Dimitrovski and Matos (2000) explain how the correlation or dependency of one variable to another (i.e. cumulative present value to years), if unknown, will relate in an incorrect resulting fuzzy number. Such an incorrect resulting fuzzy number occurs after an operation (e.g. multiplication) between non-independent fuzzy numbers of the two variables. Advantage may be taken of this fact when interpreting results that have related the two variables.

El Baradie (1995) use fuzzy logic to predict cutting speed for metal removal processes. The fuzzy sets used were triangular and hence the simplest possible. The process of simplification was exemplified by the choice of five linguistic variables representing the input variable of material hardness, and similarly the choice of five linguistic variables for cutting speed that was the output variable. Upper and lower limits were chosen for the input and output variables for the purposes of partitioning the space of each. The model greatly simplifies understanding of cutting speed by its use of words.

### **3.8.9 Cost Model Level**

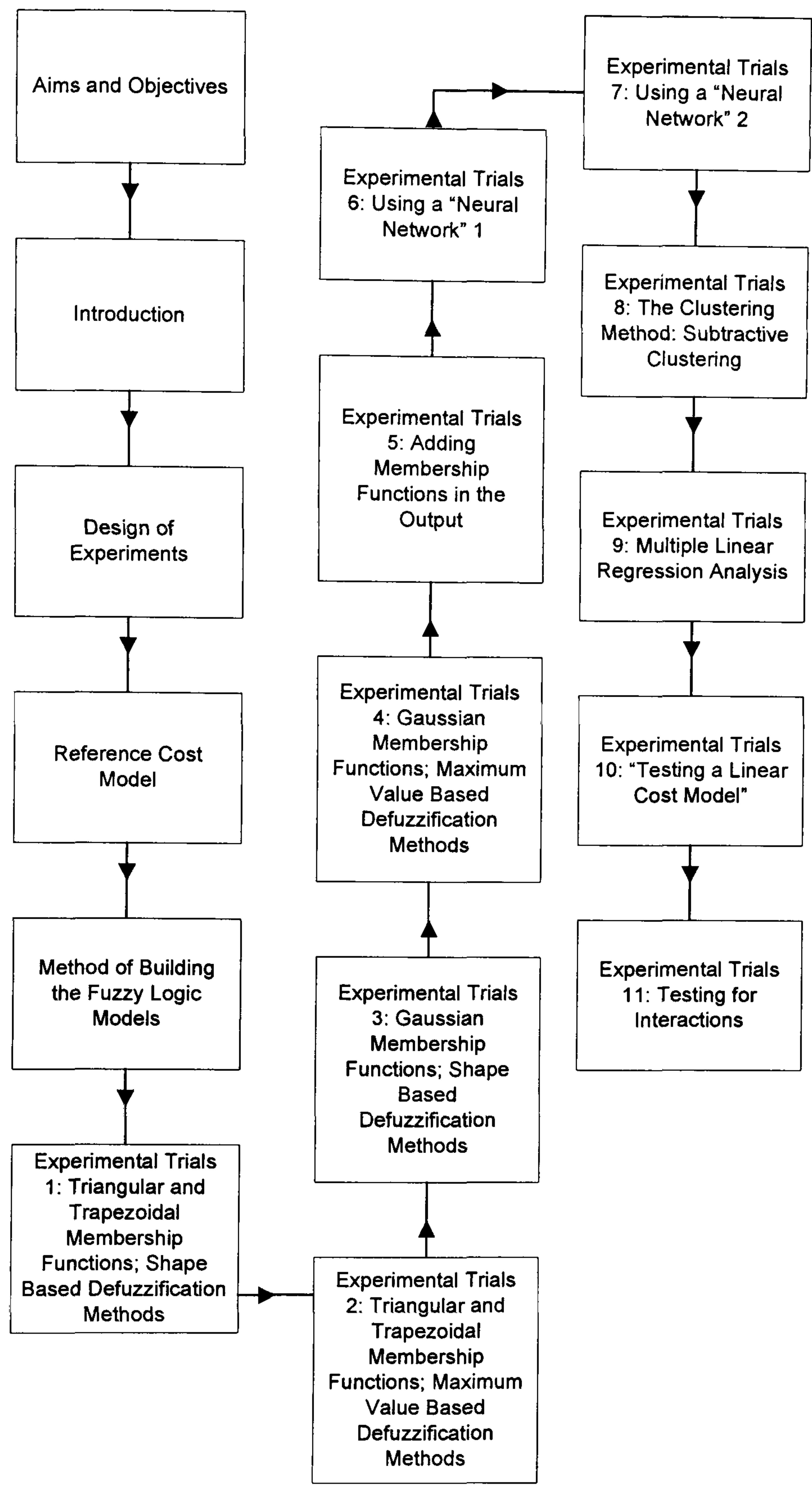
Cost models can be “high level” relating product specifications to cost, or “low level”, in which costs are built up from very detailed breakdowns of parts and activities, e.g. manufacturing activities. Two main thrusts of fuzzy logic are (1) data driven, and (2)

expert driven modelling. Data driven modelling utilises input, output data pairs such as in ANFIS (Jang 1993, Jang et al 1997, Lo 2003), whereas expert driven modelling utilises human choice of structural elements. The two approaches are complementary, though some users of data driven modelling have warned about deriving meaning from fuzzy sets in the models in which data has been used to adapt them (Mitaim and Kosko 2001). High level models are mostly associated with expert driven fuzzy logic, and low level models with data driven fuzzy logic.



# CHAPTER FOUR

## EXPERIMENTAL DESIGN



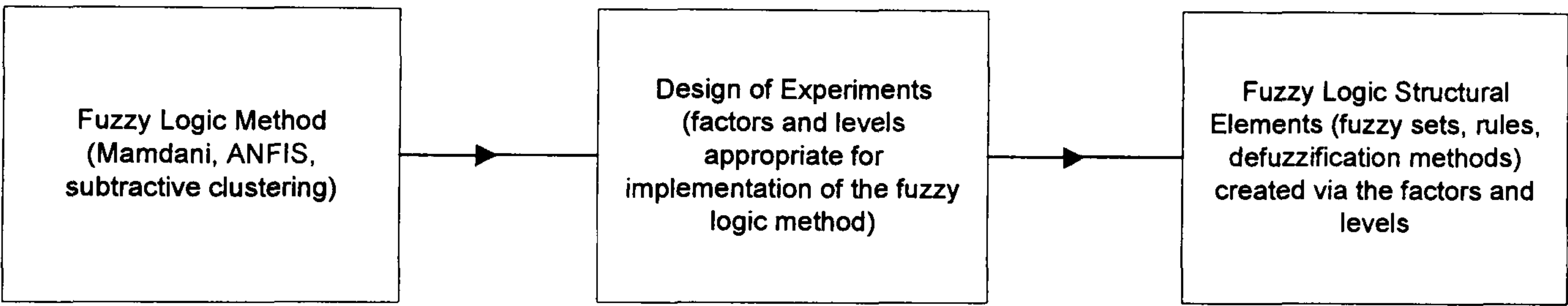
### 4.1 Introduction

This chapter describes how a choice of experimental design is made in order to efficiently test the fuzzy logic methods and their respective fuzzy logic structural

elements. Fuzzy logic is tested on two cost model forms that represent the potential complexity of cost models found in the literature, and also the potential simplicity (Dhillon 1989, Lin and Chang 2002, Ostwald 2003, Wang and Stockton 2001). The results are to be used alongside examples from the existing literature, to produce a proposed decision making methodology for the use of fuzzy logic.

A number of “design of experiments” methods were considered. The most appropriate one was chosen for exploring the space of all potential combinations of fuzzy logic structural elements efficiently. The combinations of fuzzy logic structural elements were reached via factors and levels particular to the implementation of each fuzzy logic method, as shown in Figure 4.1.

**Figure 4.1: Use of Factors and Levels to Implement Fuzzy Logic Methods.**



Therefore each fuzzy logic method (e.g. the Mamdani method) means different factors and levels are considered in a “design of experiments” approach. These factors and levels, in conjunction with the particular fuzzy logic method, provide for the different fuzzy logic structural elements (e.g. membership functions, rules or defuzzification methods). The methods, factors and levels are summarised in Table 4.1. Those particular to the Mamdani method are summarised in Table 4.2. A fuller description of the factors in Table 4.1 are found in Appendix A4.1.

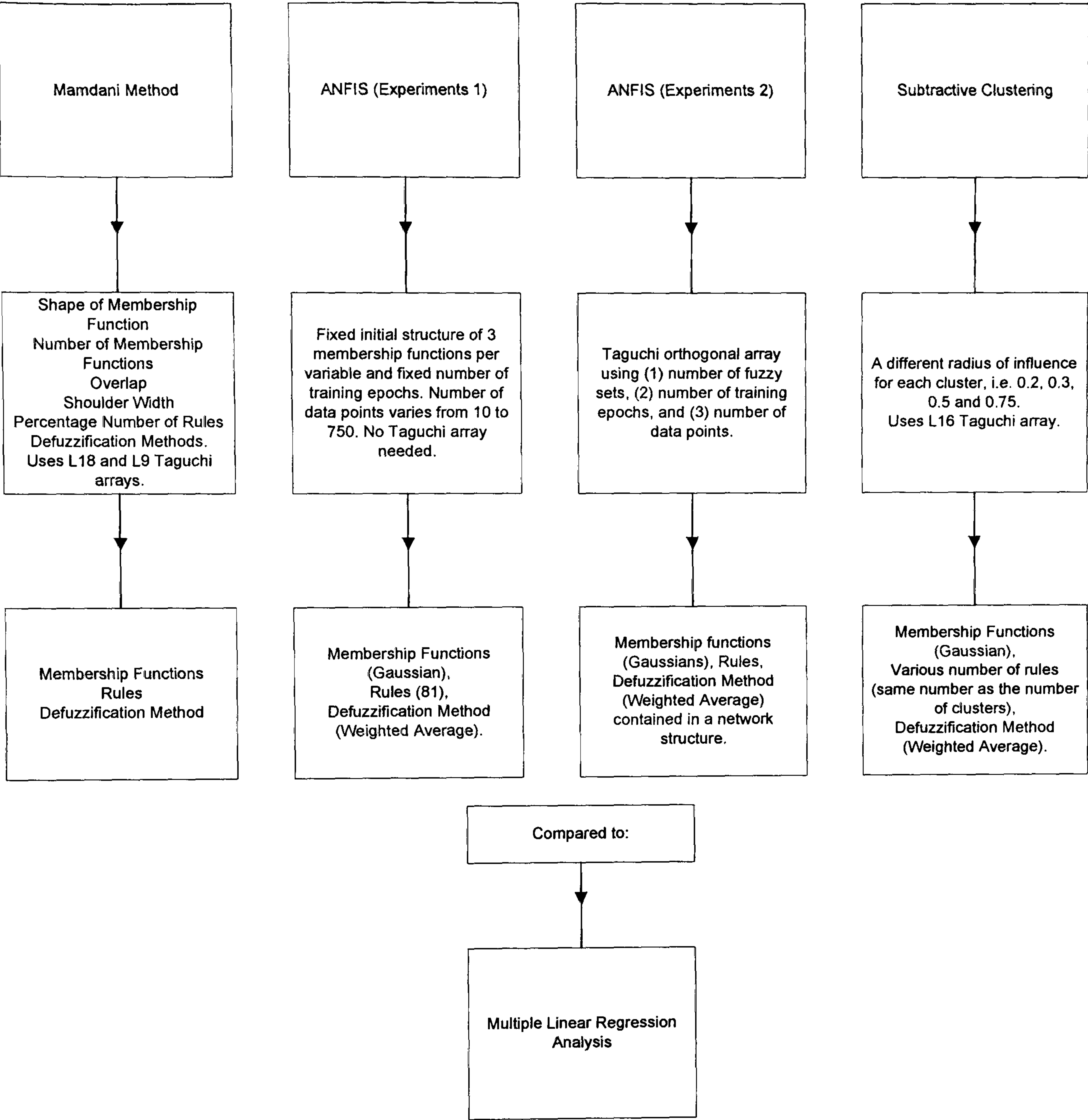


**Table 4.1: The Fuzzy Logic Methods and Their Associated Factors and Levels**  
**Used in a Design of Experiments.**

Fuzzy Logic Method	Factors	Levels
Mamdani	Type of fuzzy set	Triangular, Trapezoidal, Gaussian
	Number of fuzzy sets	3,5,7
	Percentage of the range of variable covered by overlapping fuzzy sets and distributed evenly	25, 50, 75
	Shoulder width expressed as a percentage of the width of a fuzzy set	0, 40, 75
	Percentage number of rules	100, 50, 25
	Standard deviation of a Gaussian fuzzy set expressed as a percentage of the range of the variable	75, 50, 25
	Defuzzification methods	Centroid, Bisector, Mean Of Maximum, Smallest Of Maximum, Largest Of Maximum
Adaptive Neuro-Fuzzy Inference System (ANFIS)	Number of Gaussian fuzzy sets	2, 3, 4
	Number of training epochs	50, 60, 70, 80
	Number of data points used to build the model	50, 300, 500, 750
Subtractive clustering	Radius of normalised cluster influence in each input variable X1, X2, X3, X4, Y	0.2, 0.3, 0.5 and 0.75

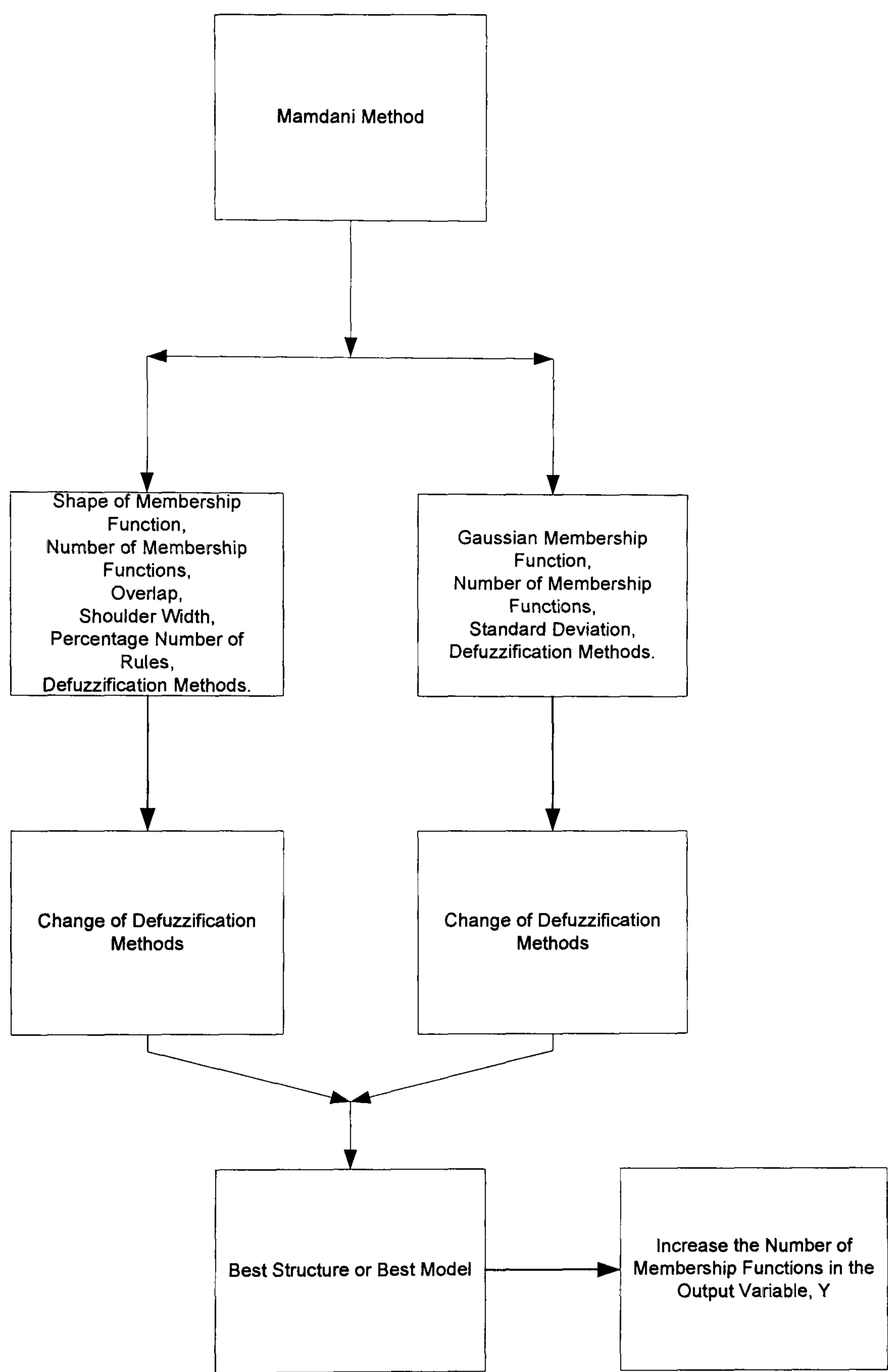
A high level overview of all the experiments conducted in the research is provided in Figures 4.2, 4.3 and 4.4. It is important to note that more thorough definitions of “percentage overlap” and “percentage number of rules” are found in Appendix A4.1.

**Figure 4.2: An Overview of All Fuzzy Logic Experiments.**

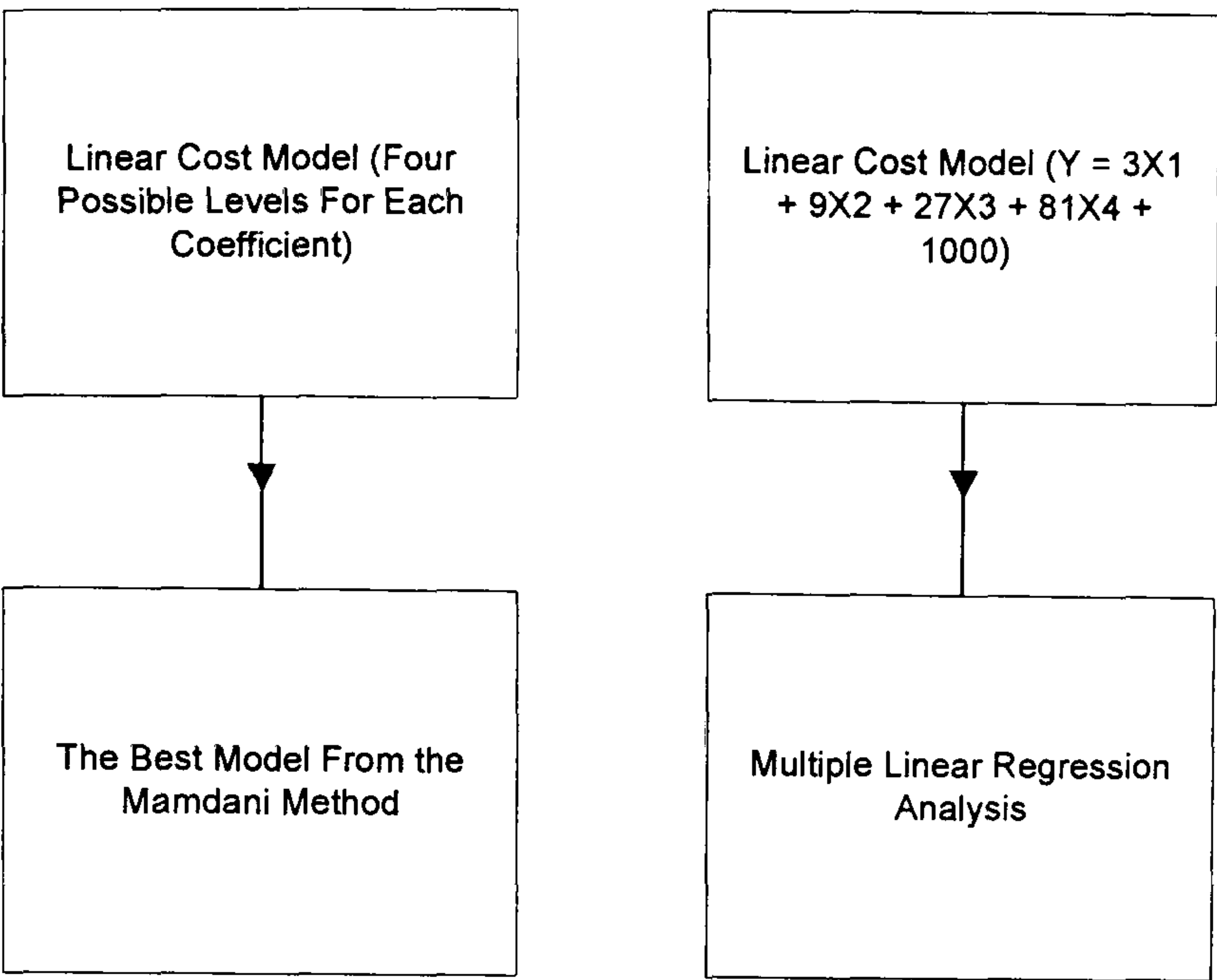




**Figure 4.3: Overview of the Mamdani Method Experiments.**



**Figure 4.4: Linear Cost Model.**



**4.2 Design of Experiments**

In order to achieve the stated objectives in an efficient and effective manner, basic methods for the “design of experiments” were compared, i.e.,

- (a) random unplanned experiments under a tight time constraint,
- (b) full factorial experiments where all combinations of all levels of all factors are considered, making for a potentially large number of experiments (Yates 37), and
- (c) use of Taguchi orthogonal arrays (Bendell et al 1989).

Random, unplanned experiments occur through the lack of a systematic approach. They do not guarantee the best solution to the problem and hence are not a robust method. A more optimum combination of fuzzy logic structural elements is missed because all combinations are not considered, especially in a short time period.



**Table 4.2: Fuzzy Logic Structural Elements for the Mamdani Method.**

- a) Shape of Membership Function, e.g. those shown in Figure 3.2.
- b) Number of membership functions, e.g. alternatives of 3, 5 and 7.
- c) Overlap of membership functions based on a uniform share of the percentage of the range of the input variable.
- d) Shoulder width of a membership function based on a percentage of the base width of the fuzzy set.
- e) Percentage of the total number of rules taken in a certain manner as described in Section 4.6.
- f) Defuzzification methods whose alternatives are the Centroid, Bisector, Mean Of Maximum, Smallest Of Maximum and Largest Of Maximum. The weighted average method is used for clustering and Adaptive Neuro-Fuzzy Inference System. See Appendix A1 for mathematical details of defuzzification methods.

A full factorial treatment of the design of experiments considers every single possible combination of levels of each factor. In this respect  $N_p$  number of experiments would have had to be completed, i.e.,

$$N_p = N_{f1} * N_{f2} * N_{f3} * \dots * N_{fn}$$

where:

$N_p$  = number of potential experiments,

$N_{f1}$  = number of levels of factor 1,

$N_{f2}$  = number of levels of factor 2,

$N_{fn}$  = number of levels of factor n, and

n = number of factors.

For the set of experiments termed, Experimental Trials 1, for the Mamdani method as shown in Table 4.1:

$N_{f1}$  = number of levels of “membership function shape” = 2,

$N_{f2}$  = number of levels of “number of membership functions” = 3,

$N_{f3}$  = number of levels of “percentage overlap” = 3,

$N_{f4}$  = number of levels of “percentage shoulder width” = 3,

$N_{f5}$  = number of levels of “percentage of rules” = 3, and

$N_{f6}$  = number of levels of “defuzzification methods” = 3.

Therefore, number of experiments needed,  $N_p = 2 \times 3^5 = 486$ .

A method that provides an equivalent systematic treatment of the problem but with less work would be a preferable solution. Such a treatment is the use of Taguchi orthogonal arrays. The Taguchi methodology uses orthogonality to reduce the potential total number of experiments needed. Consequently the 486 experiments described above are reduced to only 18 by choosing an appropriate orthogonal array. In particular the Taguchi methodology reduces  $N_p$  to:

$$(N_{f1} - 1) + (N_{f2} - 1) + \dots + (N_{fn} - 1) + 1$$

Where:

$(N_{fn} - 1)$  = Number of degrees of freedom for factor n. Degrees of freedom refers to the total number of useful ways a piece of information, for example a statistic, can be used. In particular degrees of freedom are: “the number of comparisons that need to be made without being redundant to derive a conclusion” (Peace 1993). The number of degrees of freedom of a factor is one less than its total number of levels. For the above example

$N_p$  is reduced to:



$(2-1) + (3-1) + (3-1) + (3-1) + (3-1) + (3-1)$ , i.e. 11 potential experiments. Eleven is less than eighteen so that this number of experiments can be accommodated by an  $L_{18}$  array, according to the Taguchi methodology.

The Taguchi methodology for “design of experiments” was therefore the most efficient and effective method for robust design and has been used to design a series of experiments intended to identify the best and worst fuzzy logic cost models in terms of causing least error.

### 4.3 Building the orthogonal array

The theory of Taguchi’s robust design can be found in the literature, for example Ross (1991), Peace (1993) and Bendell et al (1989). From visual inspection of the fixed orthogonal arrays from Taguchi, a suitable array for each experiment has been chosen. In constructing the experiments it was found, in addition to the “independent” effect that each alternative factor may have, that it may be necessary to consider effects arising from an interaction between factors. Interaction of one factor’s levels can occur with the levels of another (Unal et al 1993).

### 4.4 Reference Cost Model

The spreadsheet based version of the function shown in Equation (4.1) was used to generate the cost data from which (1) experimental fuzzy logic models were built, and (2) errors were calculated by comparison with actual values, i.e.:

$$Y = c + aX1 + b(X2)^d + e(X3)^f X4 \quad (4.1)$$

where :

$Y$  = cost to be calculated (£s)

$a$  = constant (5)

$b$  = constant (100)

$c$  = constant (10)

$d$  = constant (0.5)

$e$  = constant (5)

$f$  = constant (3)

$X_1$  = variable where  $1 \leq X_1 \leq 5$

$X_2$  = variable where  $100 \leq X_2 \leq 1000$

$X_3$  = variable where  $0.1 \leq X_3 \leq 0.9$

$X_4$  = variable where  $1000 \leq X_4 \leq 100000$

A possible sample of the data used is shown in Table 4.3. The numbers in columns  $X_1$ ,  $X_2$ ,  $X_3$  and  $X_4$  were generated randomly between the ranges shown. The function in equation (4.1) was used to generate  $Y$ . Equation (4.1) is made from a constant, a linear term and two non-linear terms and is relatively complex when compared to cost models in the literature (Boothroyd et al 2002, Ostwald 2003, De la Mare 82, Chen and Liu 1999). Hence use of the model provides a challenge for modelling by the fuzzy logic and Multiple Linear Regression methods.



**Table 4.3: Example of Cost Data Used to Develop Fuzzy Logic Models.**

Exp No.	X1	X2	X3	X4	Y
1	3.7	416.3	0.6	41774.9	57982.8
2	2.1	287.5	0.7	94241.6	134547.3
3	3.0	332.1	0.7	25071.5	43096.1
4	2.0	735.0	0.2	60089.1	3887.9
5	1.2	842.5	0.9	90853.4	295706.1
6	3.8	691.0	0.1	65945.6	3595.2
7	4.3	731.2	0.6	9651.8	11756.2
8	4.6	688.5	0.5	23471.2	17276.0
9	2.9	450.3	0.5	4601.1	5750.4
10	4.8	388.6	0.2	10091.2	2711.0
11	3.6	209.9	0.8	76558.1	171987.9

**4.5 Method of Building the Fuzzy Logic Models**

Equation (4.1) was used to generate data for the three different fuzzy logic methods (Mamdani (Kickert and Mamdani 1978), Adaptive Neuro-Fuzzy Inference System (Jang 1993) and the subtractive clustering based method (Chiu 1994)). The ANFIS and subtractive clustering based methods allowed a relatively straightforward input of the generated data to form a model as shown in Sections 4.6.6, 4.6.7, 4.6.8, and 4.6.11. The number of data points used to create a model was varied between 10 and 750. Thus the relationship between number of data points used and Average Percentage Error (APE), for a test data set, was examined. A separate number of data points, 350 in all, were generated and used as a test data set. Each data point from the test data set was put into the fuzzy logic model and used to calculate an output. This output was compared to the actual value Y and a percentage error calculated in all 350 cases. Hence the Average Percentage Error (APE) was calculated. The Mamdani method was not so straightforward. The steps are summarised in Appendix A4.1.

## 4.6 Description of the Experimental Trials

### 4.6.1 Experimental Trials 1: Triangular and Trapezoidal Membership

#### Functions: Shape Based Defuzzification Methods (“The Basic Experiment”)

The experimental design tables are collected together in Section A4.2. Appendix A4.1 described how equation (4.1) was used to construct fuzzy sets and rules between them for the Mamdani method. The next sections describe particular sets of experiments conducted using Taguchi orthogonal arrays. The aim of the experiments in Table A4.1 was to efficiently test combinations of factors leading to fuzzy logic structural elements, in order to find a best structure in terms of minimising an error measure.

The experiments shown in Table A4.1 were conducted. The table contains 7 columns.

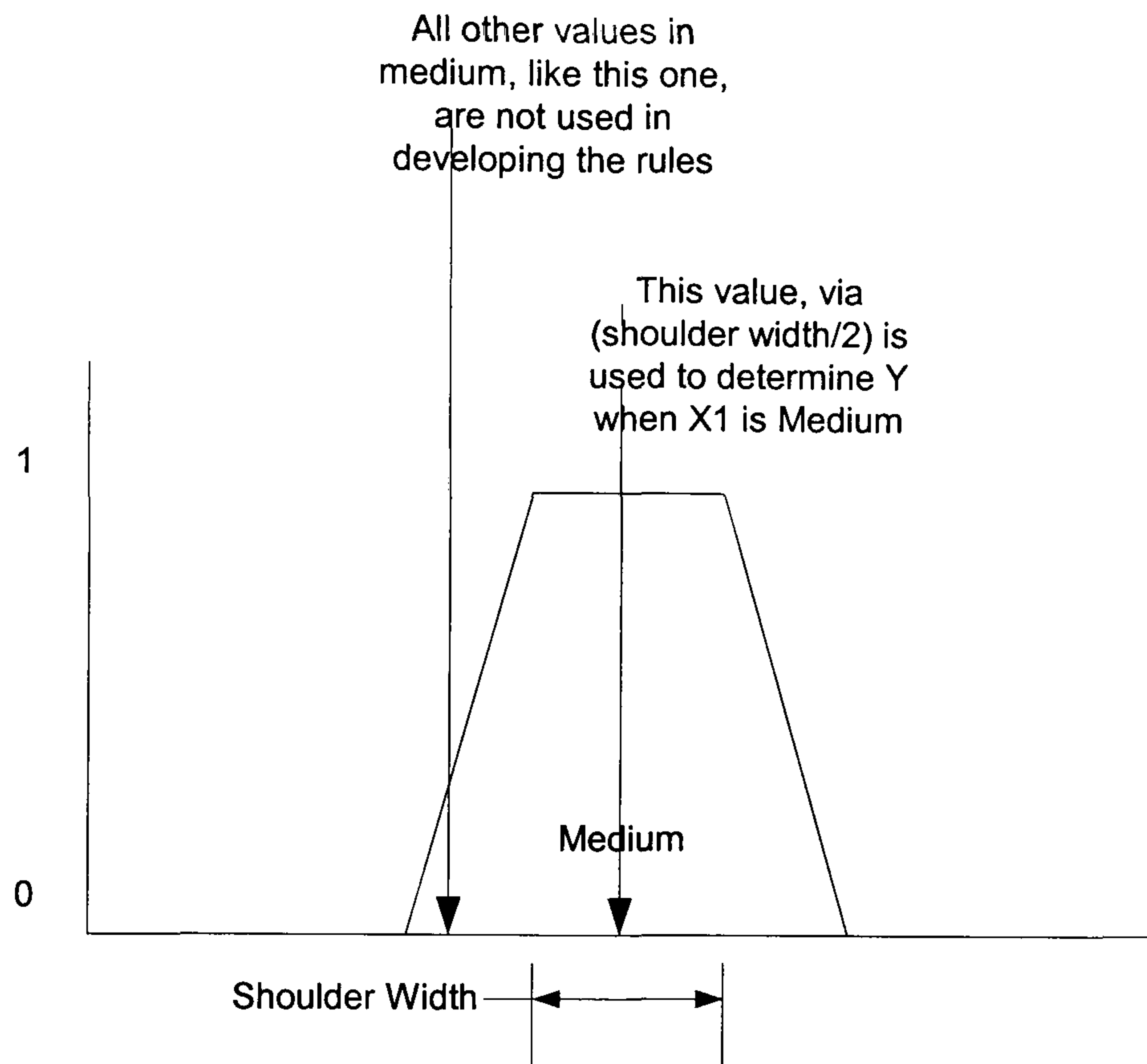
1. Column one contains the experiment number. There are 18 in an  $L_{18}$  array.
2. Column two contains details of the fuzzy set shape, i.e. triangular or trapezoidal.
3. Column three contains the number of fuzzy sets per variable, i.e. including both input and output.
4. Column four contains the percentage of the range of the variable that is covered by overlapping fuzzy sets. As detailed in Section A4.1(b) and Figure A4.2, the total range of overlap is shared equally.
5. Column five contains the percentage shoulder width as shown in Figure 4.5. For example 0% shoulder width is triangular. Fifty percent shoulder width means 50% of the base width of the fuzzy set forms a shoulder of a symmetrical trapezoid.
6. Column six contains the percentage of the complete rule base used. Fifty percent means every other rule is systematically deleted from the rule base described in



Section A4.1(c). 25% of the rule base means every other rule is systematically deleted from 50% of the rule base.

7. Column seven contains the defuzzification methods.

**Figure 4.5: Trapezoid and its Shoulder Width.**



#### **4.6.2 Experimental Trials 2: Triangular and Trapezoidal Membership Functions: Maximum Value Based Defuzzification Methods (“Alternative Defuzzification Methods”).**

The aim of this series of experiments is to examine and compare the use of additional defuzzification methods (Jang et al 1997, Roychowdhury and Pedrycz 2001). These were conducted in an attempt to improve the results of the method, i.e. the method was under development and evolving. The defuzzification methods were Mean of Maximum (MOM), Smallest of Maximum (SOM) and Largest of Maximum (LOM). In order to examine the consistency, the MOM method had been tested twice in Experimental

Trials 1 and 2. In total five defuzzification methods were tested using two sets of experiments. The experiments conducted on the Experimental Trials 1, “Basic Experiment” with the “Alternative Defuzzification Methods” are shown in Table A4.2.

#### **4.6.3 Experimental Trials 3: Gaussian Membership Functions: Shape Based Defuzzification Methods (“Using Gaussian Membership Functions”).**

The aim of this series of experiments was to examine the particular effects of using Gaussian membership functions (Demicco 2004) whilst keeping the method used in Section A4.1 for building the rules the same. Gaussian membership functions are normal distributions and were used instead of triangular and trapezoidal. The experiments carried out are shown in Table A4.3. The potential advantages of using Gaussian membership functions are: they have no discontinuities in their shape; and they are part of well known mathematical proofs (Kosko 1994), as well as other particular details, of universal function approximation. It is noted that there are other conditions not satisfied in these experiments that are necessary for universal approximation but scope for modification and further work is possible. Universal approximation is covered in Section 3.7.1 and Appendix A1 and means a method can approximate any mathematical function to an arbitrary degree of accuracy.

The Gaussian membership functions have two variable parameters: the mean, or centre of the function, and the standard deviation. The standard deviation is expressed as a percentage of the range of the variable, i.e. 25% standard deviation meant 25% of the variable range was used for the standard deviation of the Gaussian membership function.



#### **4.6.4 Experimental Trials 4: Gaussian Membership Functions: Maximum Value Based Defuzzification Methods. (“Gaussian Membership Functions” and “Alternative Defuzzification Methods”).**

The aim of the experiments was to test the Gaussian membership functions in conjunction with the alternative defuzzification methods (Jang et al 1997) (Table A4.4). The same experiments as those termed: Experimental Trials 3 (“Gaussian membership functions”) were conducted with three different defuzzification methods. These were the same as in Experimental Trials 2 (“basic experiment”) and (“alternative defuzzification methods”), i.e. MOM, SOM and LOM methods (Section 4.6.2). These experiments were subsequently termed: Experimental Trials 4 (“Gaussian Membership Functions” and “Alternative Defuzzification Methods”). It may seem inefficient in the way the experiments had been conducted, i.e. all the previous experiments can be repeated in only one larger orthogonal array. The reason was that the research progressed by an evolution of knowledge and hence some trial and error occurred. New options have been considered that make for a change in experimentation.

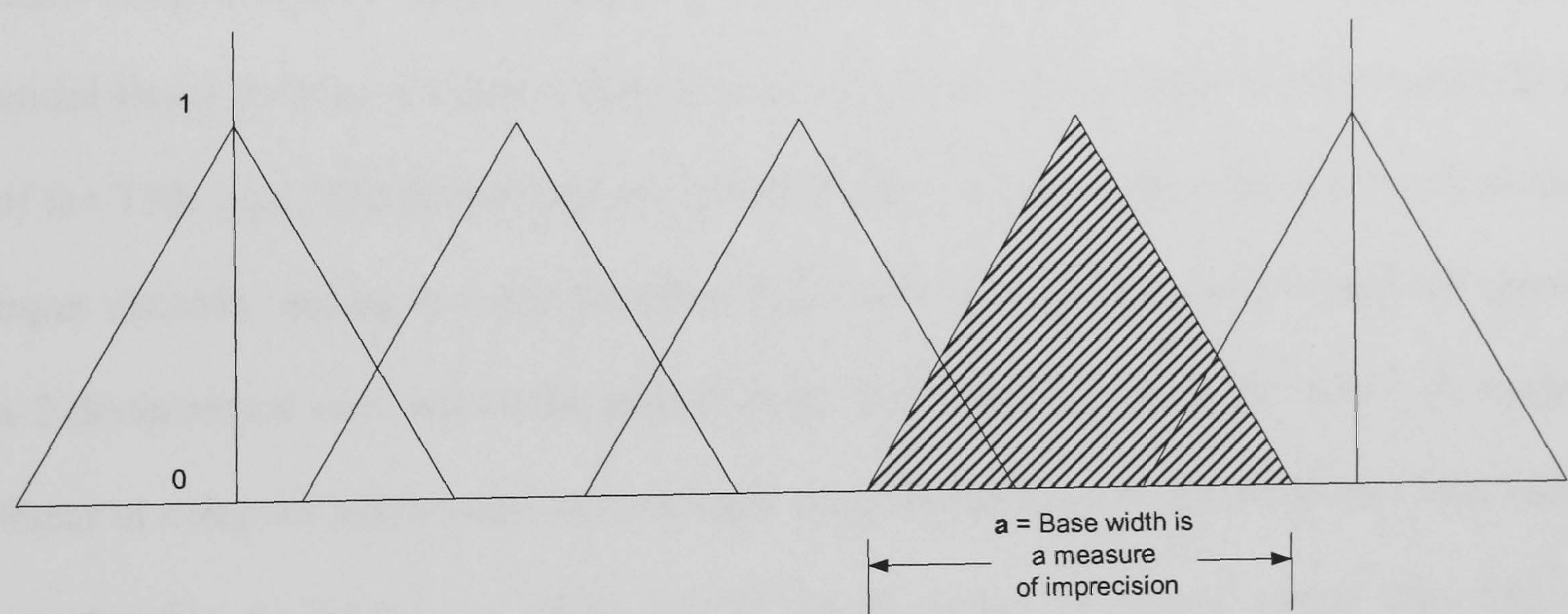
#### **4.6.5 Experimental Trials 5: “Adding Membership Functions in the Output”**

The aim of these experiments was to test a change in imprecision in the fuzzy logic models. The strategy in building less imprecision into the model, is to increase the number of membership functions in the output variable. In this way the rules combining the input variables’ fuzzy sets could choose from a larger number of membership functions in the output variable with, therefore, less base width of the fuzzy set, and therefore less imprecision. In previous experiments there was the same number of membership functions in the output variable as in each input variable. Now there would



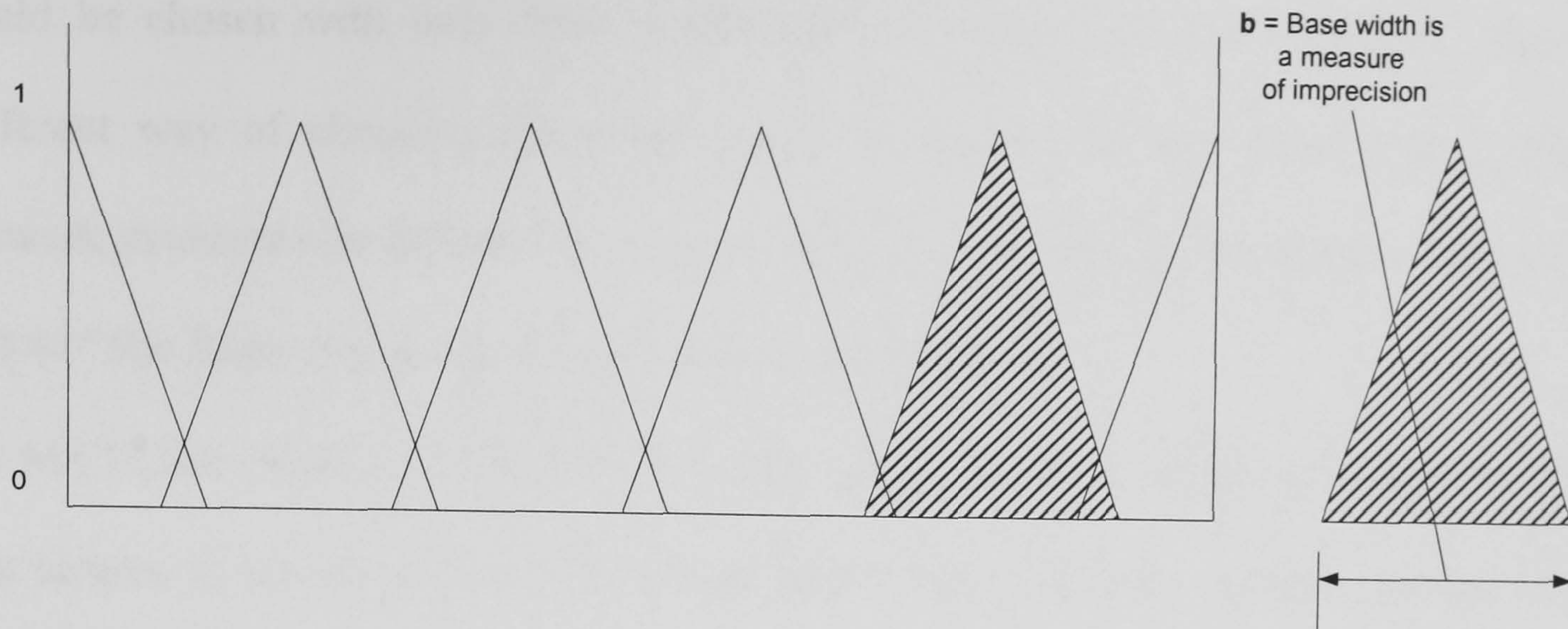
be a gradual addition of membership functions to choose from in the output variable when combining the membership functions in the input. Increasing the number of membership functions is decreasing the granularity or coarseness of the output space. The experiments carried out were with the Gaussian membership functions. Membership functions were added to the output variable using the “best model” for the cost models in Experimental Trials 4: Gaussian Membership Functions: Maximum Value Based Defuzzification Methods, i.e. cost model number 9. The process is illustrated in Figures 4.6 and 4.7 with triangular fuzzy sets. An increase in fuzzy sets in the output variable occurs from Figure 4.6 to Figure 4.7. It is noted that the imprecision,  $b$  is less for more fuzzy sets in the output variable compared to,  $a$ , that has less fuzzy sets in the output variable.

**Figure 4.6: Imprecision.**





**Figure 4.7: Increasing the Imprecision.**



#### **4.6.6 Experimental Trials 6: “Using a Neural Network” 1**

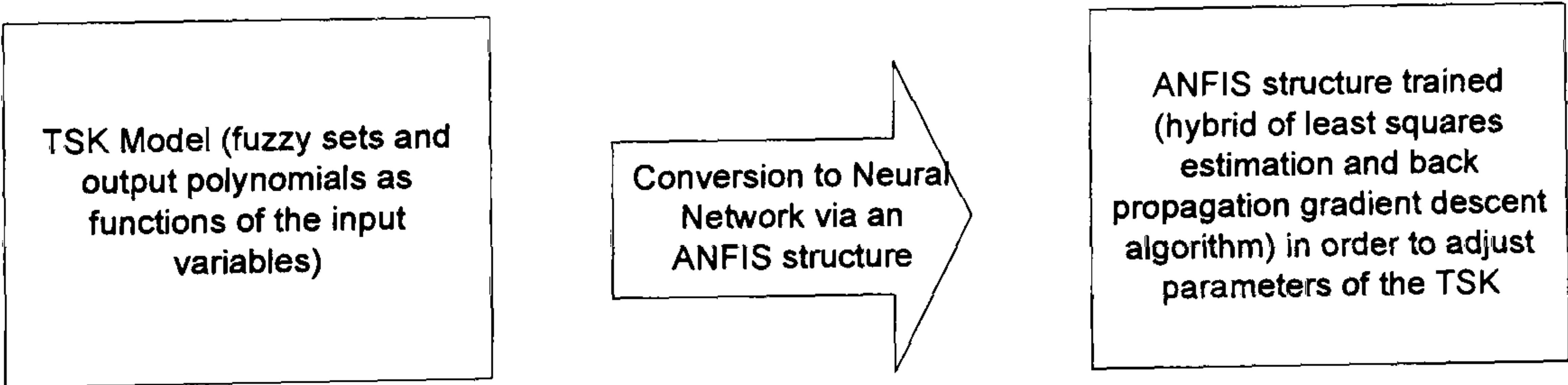
The aim of these experiments was to test the use of a “neural network”, via the ANFIS method, for minimising error in a fuzzy logic model. In particular the model was a simple one in regard to previous experiments, in that there were only 3 fuzzy sets per input variable and 81 rules. A function in MATLAB called GENFIS1 is used to build an initial Fuzzy Inference System from training data from equation (4.1). This initial FIS is of the TSK type. The parameters are such that there are three membership functions per input variable leading to a grid partition of 81 cells in the input space. Figure 4.9 shows a 2-dimensional case where the grid is only  $3 \times 3$ , making 9 cells (or rules). A unique linear or constant polynomial in the output is attributed to each combination of the input membership functions, i.e. there are  $3^4$  or 81 output linguistic terms. This FIS is subsequently trained (with a hybrid least squares and back propagation algorithm) using ANFIS (Jang et al 1997) and data from equation (4.1). It is the ANFIS trained FIS that is tested for errors. The FIS has only three membership functions per variable and hence only a maximum of eighty-one rules can potentially be created. Of course there are 81 output terms in the output variable, a large number compared to the previous



experiments. If the FIS could be trained to produce minimum error, then such a FIS could be chosen with only three membership functions per input variable. Here, a different way of choosing the membership functions has been employed. Using a network structure was different than the method employed in Experimental Trials 1, 2, 3 and 4 (“The Basic Experiment”). In summary (Figure 4.8):

- (1) MATLAB created a TSK FIS from data generated from equation (4.1). The TSK was chosen as having only 3 membership functions in the input variable that are of the Gaussian type,
- (2) the FIS formed in (1) was trained in an ANFIS architecture using data from equation (4.1). The individual experiments conducted, used a different and varying number of data points in the training data set randomly generated from equation (4.1), and
- (3) each FIS was tested with a test data set of 350 data points in order to calculate the APE. The structure of an ANFIS model is shown in Appendix A1.

**Figure 4.8: Summary of GENFIS then ANFIS.**



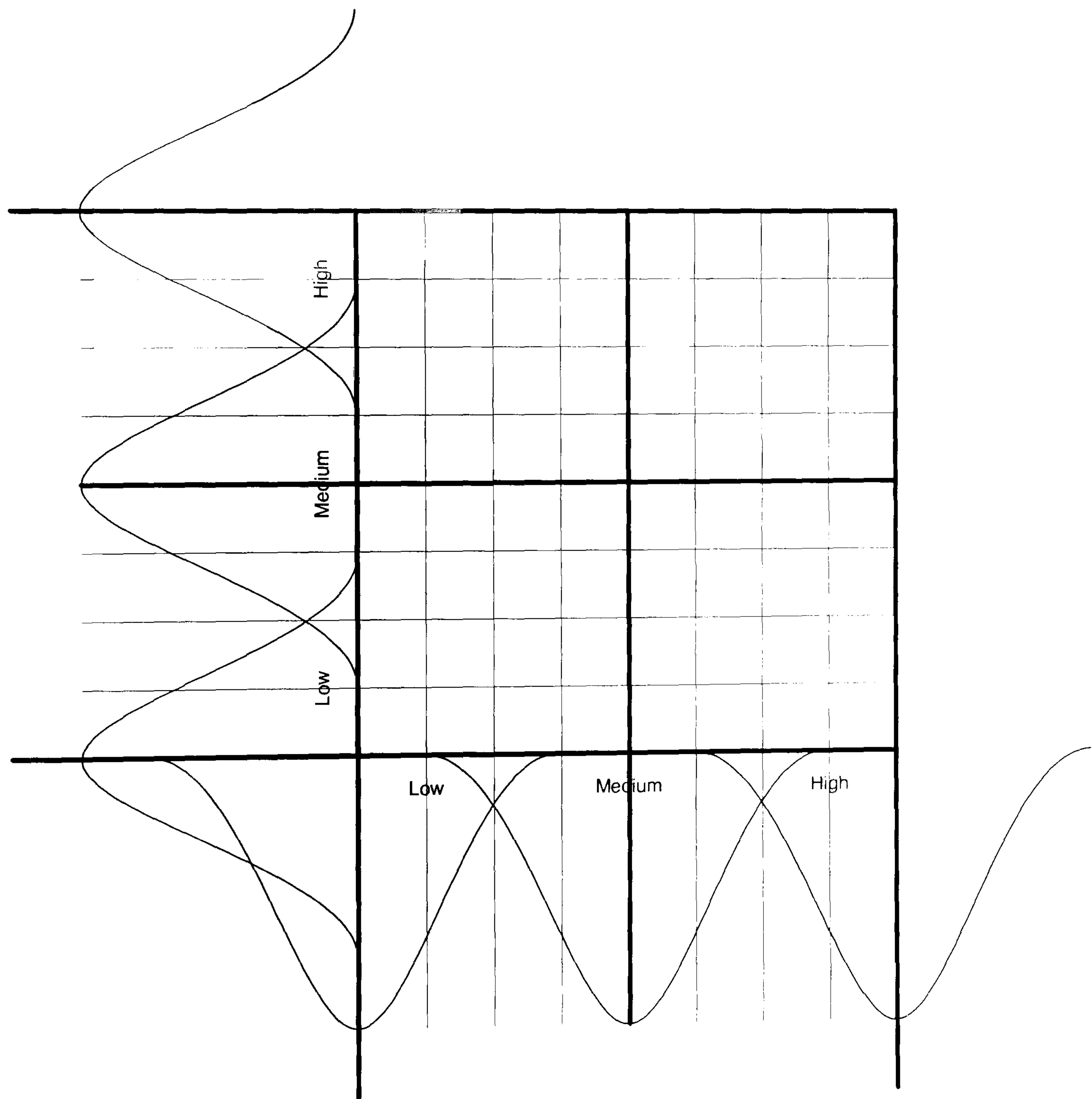
#### 4.6.7 Experimental Trials 7: “Using a Neural Network” 2

Having used a network structure to show how accurate a simply structured Fuzzy Inference System (FIS) can be made, a second set of experiments, “Using a Neural Network 2” were carried out as shown in Table A4.5.



Table A4.5 tested the ANFIS method for estimating Equation (4.1) under a wide range of conditions in an efficient manner. The number of fuzzy sets varied as 2, 3 and 4 fuzzy sets. The number of epochs varied as 50, 300, 500, and 750. The number of data points varied as 50, 300, 500 and 750. Again the hybrid least squares and back propagation algorithm was used for training.

**Figure 4.9: GENFIS1 Builds an Initial FIS.**



#### 4.6.8 Experimental Trials 8: “The Clustering Method: Subtractive Clustering”

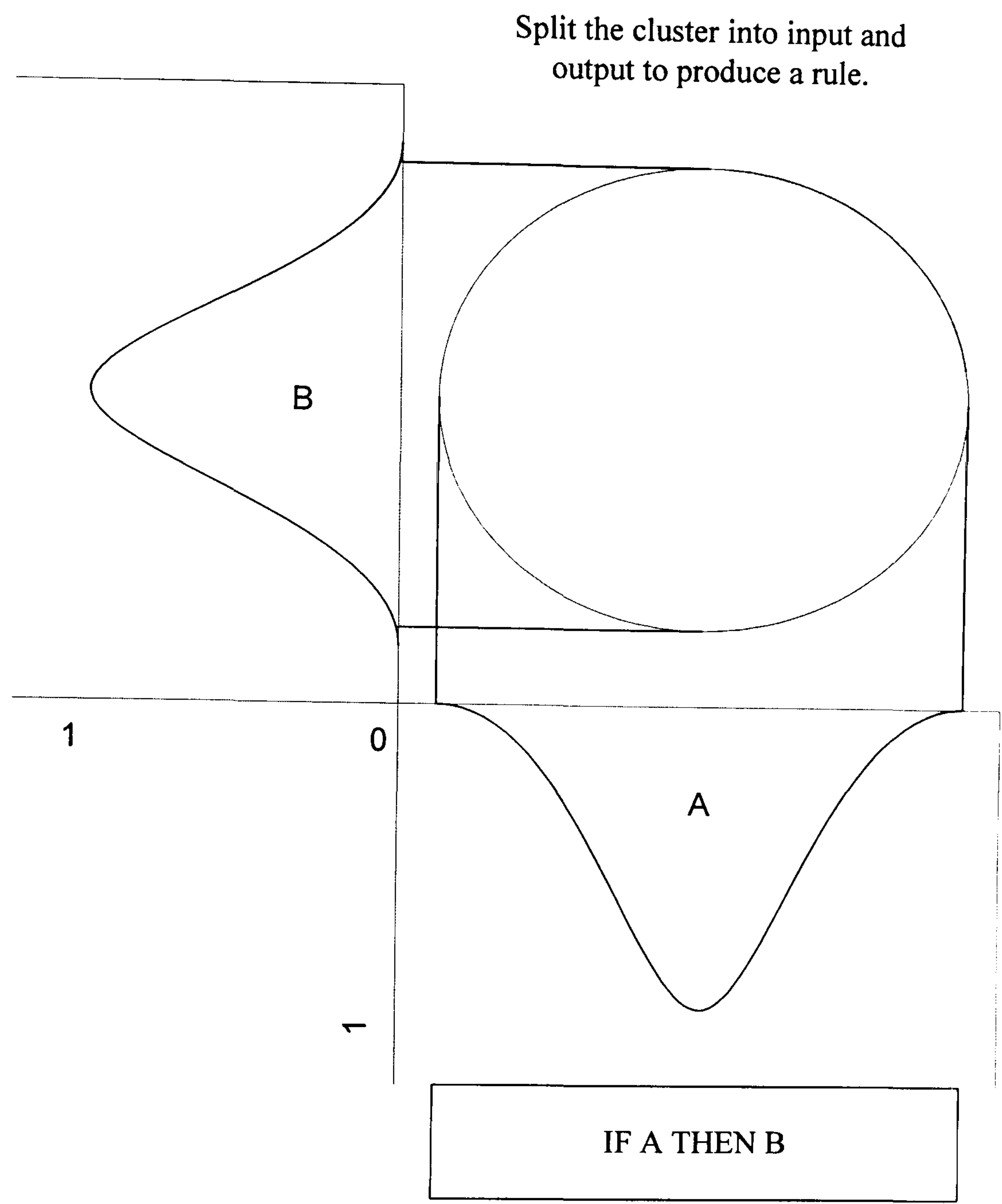
The aim of this series of experiments was to test the efficacy of subtractive clustering, as identified in Chapter 3, for estimating equation (4.1). In particular the experiments were carried out to give insight into the performance of the method when varying parameters affecting a property called, “cluster influence”. A function in MATLAB called GENFIS2 uses subtractive clustering in order to build a FIS. Subtractive clustering clusters the data based on an algorithm introduced by Chiu (Chiu 1994). The algorithm assigns degrees of membership to fuzzy sets to the individual data points based on a distance metric and cluster centres. The L16 Taguchi orthogonal array is used to choose the parameters of the clustering method.

An L16 was used to choose the parameters for the GENFIS2 algorithm as seen in Table A4.6. GENFIS2 accepted normalised input and output vectors of a training set as arguments as well as 4 normalised parameters, one for each input variable. The parameters are labelled specifically with letters A (0.2), B (0.3), C (0.5) and D (0.75) that are values on a scale between 0 and 1. These determine the radius of influence of cluster centres for  $X_1$ ,  $X_2$ ,  $X_3$ ,  $X_4$  and  $Y$ . The algorithm determines the input and output fuzzy sets and the rules from a set of training data including inputs and outputs and this radius of influence. The distance of input values to the input part of the cluster centres is used as the basis of calculating firing strengths of rules. The output parts of the rules are assumed to be linear functions of the output variables, i.e. a TSK structure. The consequents in the rules are subsequently determined by linear least squares estimation from training data.



The clustering algorithm produces several rules and membership functions. In particular a cluster is split into its input and output dimensions in order to determine a rule. Figure 4.10 shows an example of this splitting for a cluster consisting of a 2 dimensional cluster that is split into a 1 dimensional input and a 1 dimensional output only. As explained earlier, the output part of the rule is assumed to follow a TSK structure. In particular the “output fuzzy set” in Figure 4.10 is discarded for an assumed local linear function of the input variables. The parameters of each local output linear function are determined by linear least squares estimation from the training data. This method differs from Experimental Trials 1 to 4 (“The Basic Experiment”) in that each membership function is used only once. In particular any fuzzy set from an input variable appears in one rule only.

**Figure 4.10: A Cluster Forms a Rule.**



**4.6.9 Experimental Trials 9: Multiple Linear Regression Analysis**

The aim of this series of experiments is to test the ability of Multiple Linear Regression (MLR) analysis to approximate equation (4.1) via the data generated data from it. The number of data points was varied from 10 to 750 and the effect on APE of the MLR analysis noted. MLR was chosen for comparison purposes because of its popularity in cost modelling (Roy et al 2001, Kelly et al 1995, Ostwald and McClaren 2004).



#### 4.6.10 Experimental Trials 10: “Testing a Linear Cost Model”

The aim of this series of experiments is to test the ability of the Mamdani method and then MLR analysis to approximate a linear function. In particular the function shown in Equation (4.2) was used to generate the cost data from which the cost model with the least error from Experimental Trials 1-4, was tested. Therefore cost model 9 or the “best model” from Experimental Trials 4, from the Mamdani method, was tested for its ability to minimise APE with 350 test data points. As shown in Equation (4.2) the function has coefficients A, B, C, and D. Each of these coefficients takes on the values 3, 9, 27 or 81, as decided by the Taguchi L16 orthogonal array in Table A4.7, with the four factors of X1, X2, X3 and X4 with these four levels each.

$$Y = AX1 + BX2 + CX3 + DX4 \quad (4.2)$$

The function shown in equation (4.3) was used to generate the cost data from which the MLR analysis was tested for its ability to minimise APE with 350 test data points. The number of data points used to generate the MLR model was varied in stages between 10 data points and 750.

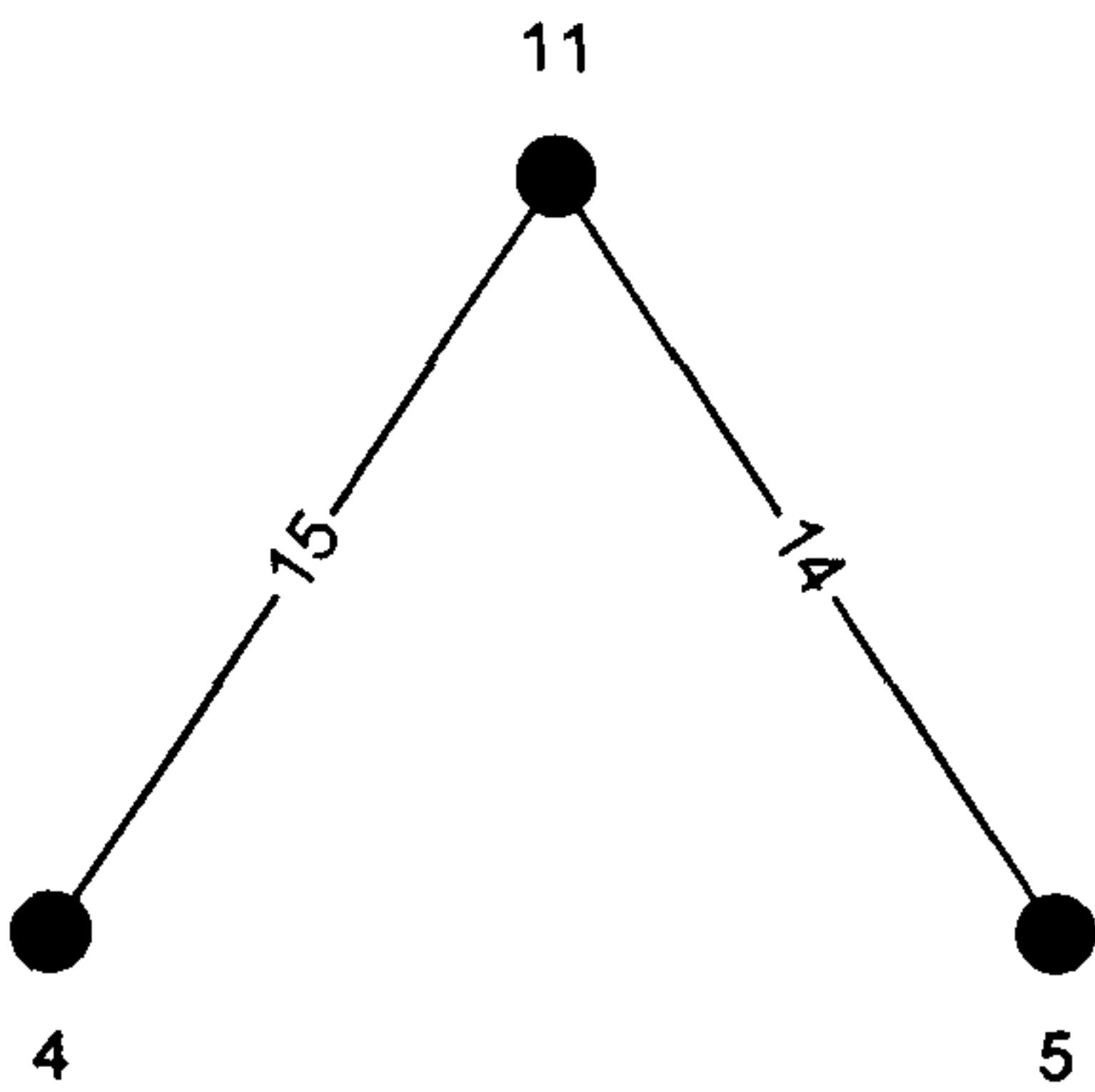
$$Y = 3X1 + 9X2 + 27X3 + 81X4 \quad (4.3)$$

#### 4.6.11 Experimental Trials 11: “Testing for Interactions”.

It was found that in some of the experiments a “best model” was apparent that outperformed the “best structure” found by the Taguchi analysis. The reason for this is proposed to be the presence of interactions between factors within the experiments.

These interactions are further discussed in Chapter 6, Section A6.2. The aim of these experiments is to investigate the presence of interactions, using linear graphs to identify in which columns of a Taguchi orthogonal array the effect of particular interactions are found. A full treatment of using linear graphs is found in Peace (1993). A simple example of one is found in Figure 4.11.

**Figure 4.11: An Example Linear Graph.**



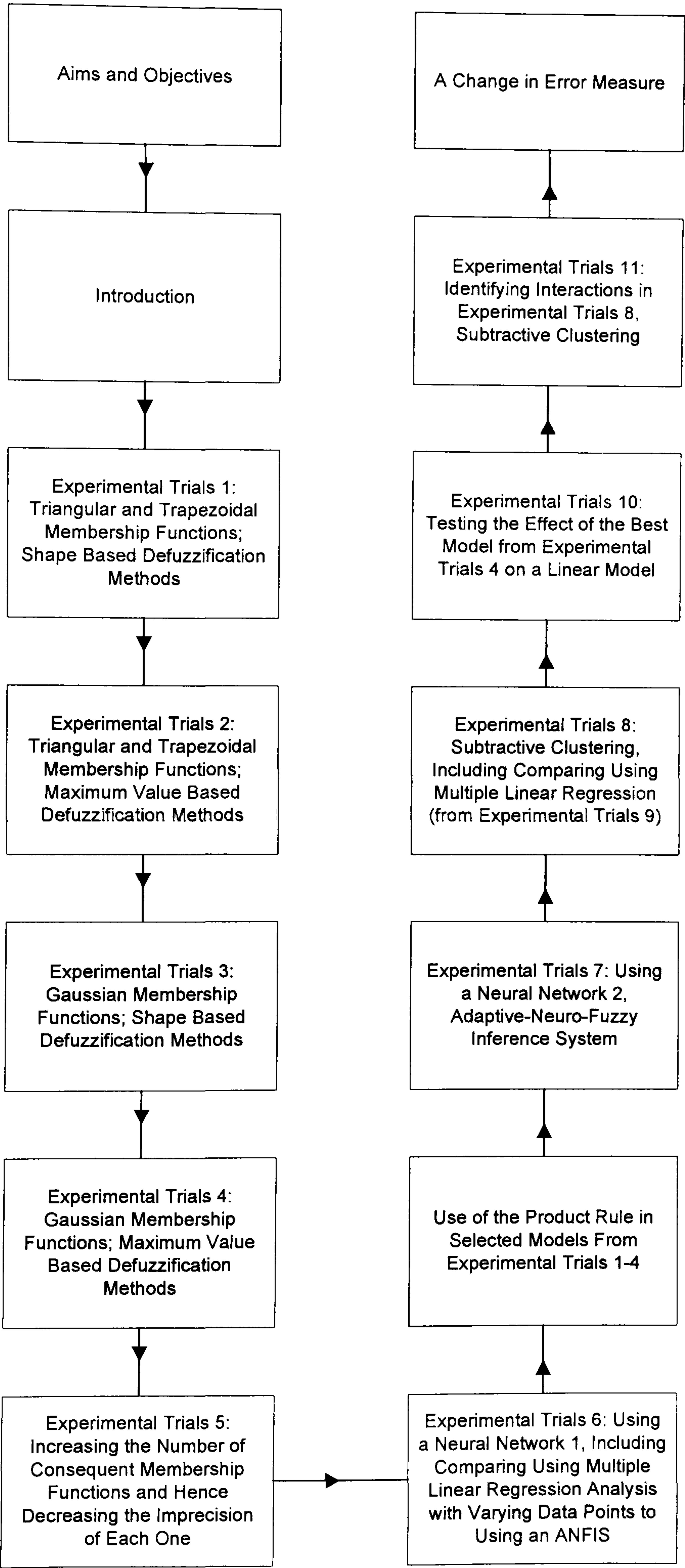
The linear graph in Figure 4.11 shows that interactions between factors in columns 4 and 11 can be found in column 15, and interactions between factors in columns 11 and 5 can be found in column 14.

Table A4.8 shows an  $L_{16}$  orthogonal array to test for the presence of interactions in the previous subtractive clustering experiments in Table A4.6. A linear graph is used to identify columns holding the interactions. The experiments in Table A4.8 differ from those in Table A4.6 in that only 2 levels of normalised cluster influence (level 1 is 0.2, and level 2 is 0.75) are used to make the process of looking for interactions easier. The process of choosing an orthogonal array for interactions must include the degrees of freedom for each interaction.



CHAPTER FIVE

EXPERIMENTAL TRIALS RESULTS



## 5.1 Introduction

The results of experiments described in Chapter 4 are presented in sequence. Points of importance are stated to be further discussed in Chapter 6. The results are presented through a series of tables and graphs as set out below.

- (1) Tables of all cost models for each set of experimental trials. The cost models are listed individually with associated Average Percentage Error (APE), (Tables A5.1, A5.5, A5.7, A5.9, A5.12, A5.15, A5.18, A5.19, A5.20, A5.21, A5.23).
- (2) Tables of structural elements listed with associated trials and the Mean Average Percentage Error, MAPE for each structural element, e.g. 3 fuzzy sets. The Tables are: (Tables A5.2, A5.6, A5.8, A5.10, A5.13, A5.16, A5.22)
- (3) Tables of different runs of experimental trials, i.e. each experiment in the array is repeated. For each run the best structure, the worst structure, the best model, the worst model, and the overall Mean APE of all the experiments are recorded (Tables A5.4, A5.11, A5.17).
- (4) Graphs recording the MAPE against each associated fuzzy logic structural element (Figures A5.1, A5.2, A5.3). These present a visual appreciation of the relative effect of the levels of each fuzzy logic structural element factor.
- (5) Graphs recording APE against an important variable, e.g. number of data points, number of rules, or number of fuzzy sets (Figures A5.5, A5.6, A5.7, A5.8, A5.9, A5.10, A5.11, A5.12, A5.13).



## 5.2 Experiment Trials 1: Triangular and Trapezoidal Membership Functions: Shape Based Defuzzification Methods

The results of Experimental Trial 1 were recorded in Tables A5.1, A5.2, A5.3, A5.4 and in Figure A5.1. In Table A5.4 the best structure was 99.7 % APE in run 1 and 93.7% APE in run 2. The worst structure was 931.3% APE in run 1 and 982.2% APE in run 2. It is noticed that one of the cost models has a lower APE than the “best structure” predicted from Table A5.2. This has been termed “the best model”. The best model in run 1 has 86.2% APE and 79.5% APE in run 2. There was also a “worst model”, worse than the “worst structure”. The worst model in run 1 has 1077.6% APE and 1107.4% APE in run 2. The results for “Experimental Trials 1” are in general poor, so much so that the model could not be used in estimating manufacturing costs or process time within the three categories of Rough Order of Magnitude, budget or definitive (as defined in Section 2.3).

Table A5.2 and Figure A5.1 provide information on relative effects of fuzzy logic structural elements by considering their Mean Average Percentage Error. It can be noticed that:

- (1) increasing the number of fuzzy sets from 3 to 7, decreases the MAPE from 610.1% to 292.5%,
- (2) increasing the percentage overlap between fuzzy sets from 25% to 75% decreases the MAPE from 577.3% to 345.5%,
- (3) there is no clear trend from increasing the percentage shoulder width of a fuzzy set, although it is noticed that the lowest MAPE is achieved at the highest percentage shoulder width used, i.e. 70%,

- (4) increasing the percentage number of the total possible number of rules used from 25% to 100% decreases the MAPE from 694.7% to 269.7%, and
- (5) the shape based defuzzification methods caused a greater MAPE than the maximum based defuzzification method used, i.e. 480.5% and 542.4% compared to 345.0%.

The relative effects of the structural elements as defined by the Taguchi Methodology are shown in Figure A5.1.

### 5.3 Experimental Trials 2: Triangular and Trapezoidal Membership Functions: Maximum Value Based

Experimental Trials 2 used only 1 run. In particular the use of maximum value based defuzzification methods led to improved results for the best structure and the best model, i.e. the best structure has 54.1% APE and the best model has 64.6% APE as shown in Table A5.5. In experimental trials 2 the best structure is indeed now the best, but the best model has improved on the best model in Experimental Trials 1 (i.e. 64.6% compared to 79.5% APE). The worst structure was 1251.6% APE and the worst model was 1707.9% APE, both worse than their counterparts in Experimental Trials 1.

Table A5.6 shows that:

- (1) there is general decrease in MAPE from 694.1% to 276.6% with an increase in 3 fuzzy sets to 7,
- (2) there is a decrease in MAPE from a high 851.1% to 337.8% with an increase in overlap from 25% to 75% of the variable range,
- (3) there is an increase in MAPE from 452.1% to 580.6% with an increase in shoulder width from 0% to 75%,



- (4) there is a decrease in MAPE from 727.4% to 386.0% with an increase in the percentage number of rules used (as described in Section A4.1) from 25% to 100%, and
- (5) the Smallest of Maximum and the Mean of Maximum greatly outperformed the Largest of Maximum defuzzification methods.

Figure A5.2 compares the MAPE for the Centroid and Bisector methods from Experimental Trials 1, Table A5.2, and the MAPE for the LOM, SOM and MOM methods in Experimental Trials 2. SOM and MOM outperform the worst of all five, i.e. the LOM, and the shape-based methods, Centroid and Bisector, in these experiments.

#### **5.4 Experimental Trials 3: Gaussian Membership Functions: Shape Based Defuzzification Methods.**

Table A5.7 shows the results of individual cost models when using Gaussian fuzzy sets and shape based defuzzification methods. It is striking that the best structure is 349.5% APE and the best model, model number 7, is 162.5% APE. Some of the cost models are over 1000% APE, i.e. number 2 (1196% APE), number 6 (1547% APE), number 8 (1636% APE), number 9 (1659% APE), and the worst structure (1623% APE).

Table A5.8 and Figure A5.3 show that:

- (1) an increase in the number of fuzzy sets from 3 to 7 causes a general increase in MAPE of 774.9% to 1127.9%,

- (2) an increase in the standard deviation of the Gaussian fuzzy set from 25% to 75% of the variable range, causes a general increase in MAPE from 573.2% to 1162.8%, and
- (3) shape based defuzzification methods showed a higher MAPE (1213% for bisector and 1344% for centroid) than the Mean of Maximum (177.7%).

**5.5 Experimental Trials 4. Gaussian Membership Functions: Maximum Value Based Defuzzification**

The use of only maximum based defuzzification methods in Experimental Trials 4 showed a great improvement over Experimental Trials 3, as shown in Table A5.9. Cost Model Number 9 has a 56.5% APE, that is lower than 58.9% APE of the best structure.

Table A5.10 shows a different trend than seen in Experimental Trials 3:

- (1) as the number of fuzzy sets increases from 3 to 7 then the MAPE decreases from 313.4% to 107.3%,
- (2) there is no general trend from increasing the Percentage Standard Deviation of each Gaussian fuzzy set from 25% to 75% of the variable range. The best MAPE was 50% of the variable range as standard deviation, at 139.1%, and
- (3) the Smallest of Maximum Defuzzification method produced the best MAPE of the maximum based defuzzification methods, i.e. 65.4%.

Figure A5.4. compares the MAPE for the shape based defuzzification methods in Experimental Trials 3 (centroid and bisector) to the MAPE of the maximum based defuzzification methods in Experimental trials 4 (LOM, MOM and SOM). There is a

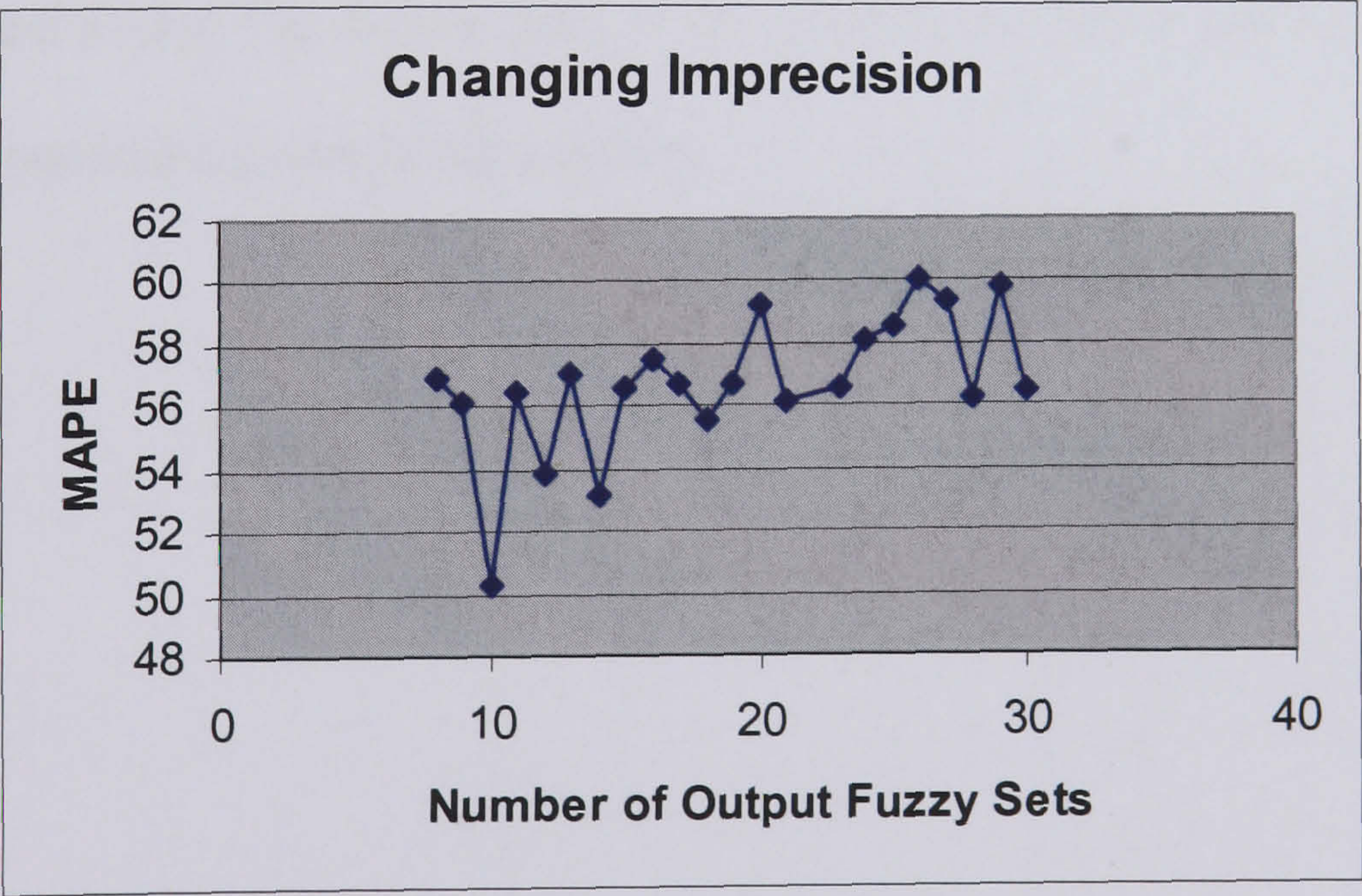


relatively large difference between the average MAPE of shape based and maximum based defuzzification methods. The difference in trends between Experimental Trials 3 and 4 infer interactions between levels of factors.

**5.6 Experimental Trials 5. Increasing the Number of Consequent Membership Functions and Hence Decreasing the Imprecision of Each One.**

Figure 5.1 shows the effect of increasing the number of outputs, or consequent fuzzy sets, for the rules to choose from, for the best model in Table A5.9. In particular a rule in the best model from Experimental Trials 4, is given more output fuzzy sets over the range of Y to choose from (in fact between 8 and 31 in 24 experiments). It is shown that there is no clear trend relating the APE and the number of consequent fuzzy sets. In fact there seems to be a threshold beyond which the best model structure cannot improve by adding the output sets.

**Figure 5.1: Gaussian Membership Functions with Decreasing Imprecision.**



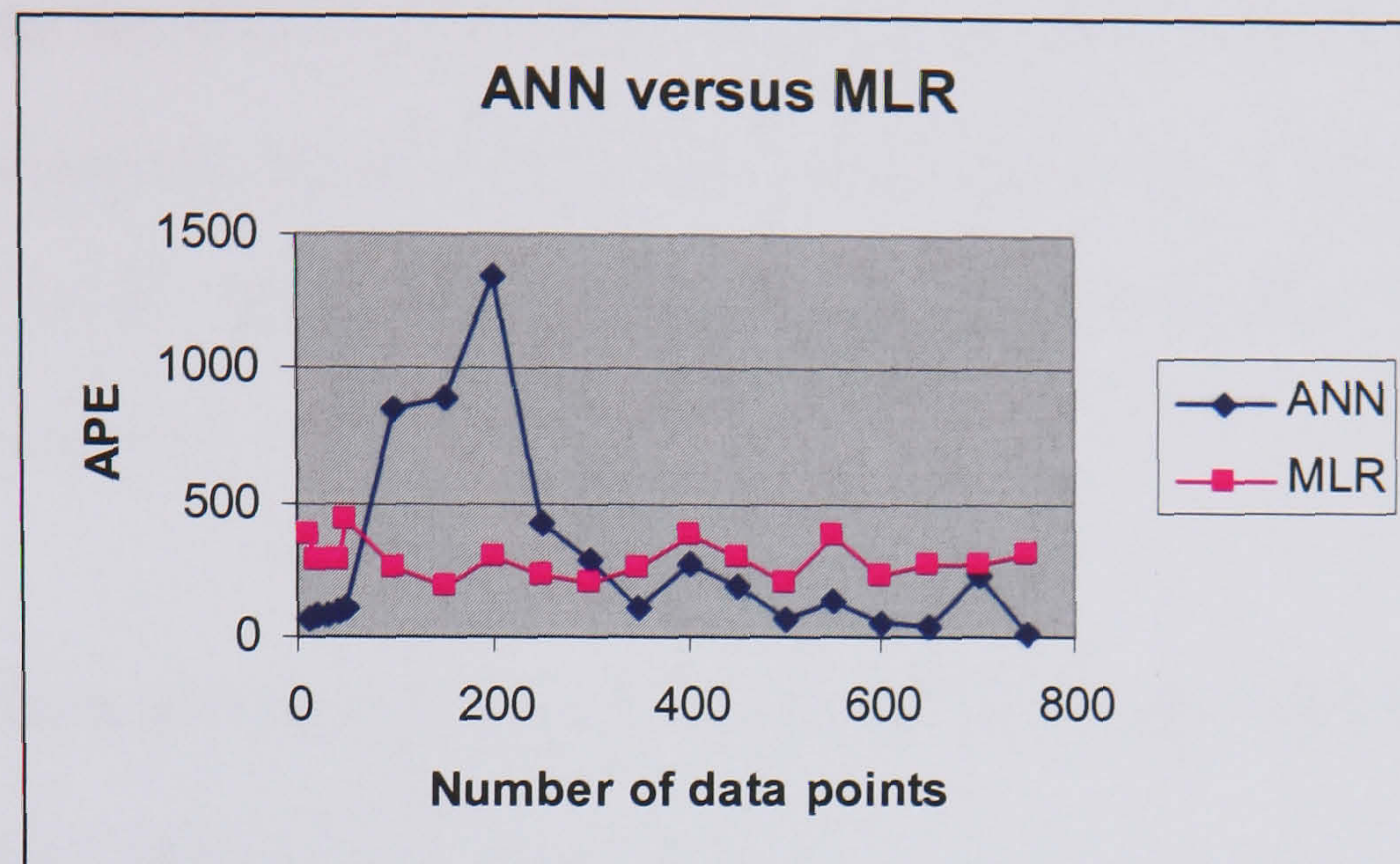


## **5.7 Experimental Trials 6. Using a “Neural Network” 1, Including Comparing Using Multiple Linear Regression (MLR) (from Experimental Trials 9) with varying Data Points to Using an Adaptive-Neuro Fuzzy Inference System (ANFIS)**

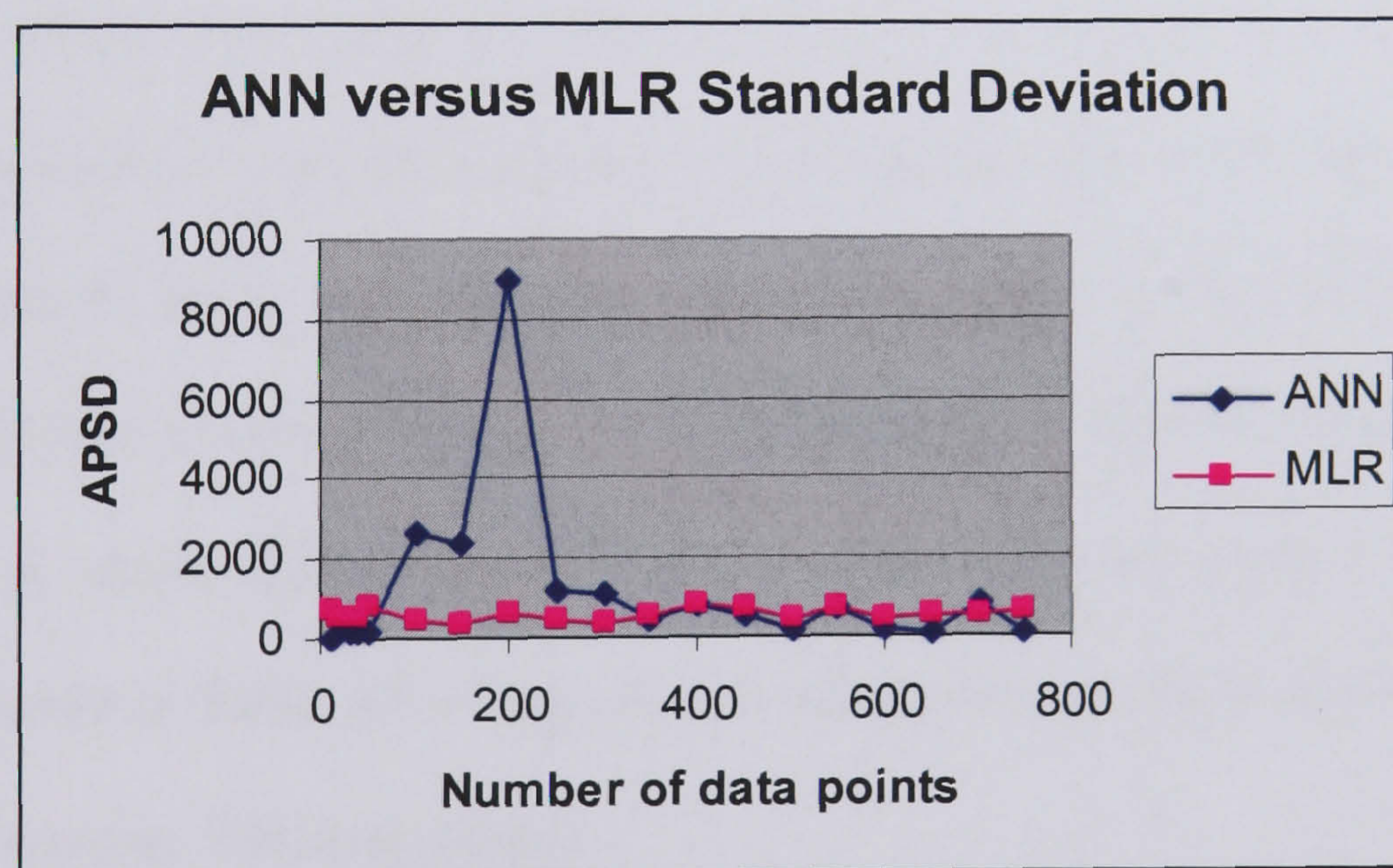
Figure 5.2 compares the use of Multiple Linear Regression and an Adaptive-Neuro-Fuzzy Inference System over the number of data points used to build both types of model. The ANFIS model used a fixed number of fuzzy sets per input variable (i.e. 3), hence a fixed number of rules (81), a fixed type of fuzzy set (Gaussian) and a fixed number of training epochs (20). The MLR results improve from 288.6% APE to 240.2% APE from using 20 data points to using 250 data points but no real trend is visible. As in Figure 5.1 a threshold seems to have been reached beyond which no improvement is possible by adding more data points. There is a clear trend for the ANFIS from 1348.9% APE at 200 data points to 15.9% APE at 750 data points. It is noticed that ANFIS can produce very poor results when very good results are expected, i.e. 231.6% APE at 700 data points. This result is sandwiched by 40.7% APE and 15.9% APE at 650 and 750 data points respectively. The best results of the ANFIS method outperformed the MLR method at the required number of data points.



**Figure 5.2: Gaussian Membership Functions With Adaptive Neuro-Fuzzy Inference System (ANFIS) Defuzzification Compared to Regression Analysis via Average Percentage Error**



**Figure 5.3: Gaussian Membership Functions with ANFIS Defuzzification Compared to Regression Analysis via Average Percentage Standard Deviation (APSD)**





**5.8 Use of the Product Rule in Selected Models from Experimental Trials 1-4.**

Table A5.11 shows the result of applying the product rule to best structures and best models from Experimental Trials 1-4. The product rule is explained in Appendix A1, and was used for the AND or implication operators or both. The product rule improved all models, for example the best model in Experimental Trials 1 was improved from 86.2% APE to 54.7% APE by using the product rule for both the AND connective and the implication operator.

**5.9 Experimental Trials 7: Using a “Neural Network” 2, Adaptive-Neuro-Fuzzy Inference System.**

The results of experimental trials 7 are shown in Table A5.12. There are a range of results from very poor (4041.2% APE in model number 9) to very good (10.8% APE in model number 3). A significant result was the obtaining of 10.8% APE with only 2 fuzzy sets per variable and hence 16 rules. Of more significance was the 1.6% APE (with 3.2% Average Percentage Standard Deviation) using the same model structure (but 700 data points to train the network). Although the epoch and data points play a part, the number of fuzzy sets and rules remains the same in this latter example. The best model and best structure both had 2 fuzzy sets and 70 epochs when the number of data points was varied (Figure 5.4). It is noticeable that the relative MAPE for the structural elements in Table A5.13 are poor in comparison to the best model attained (2 fuzzy sets, 70 epochs, 700 data points).

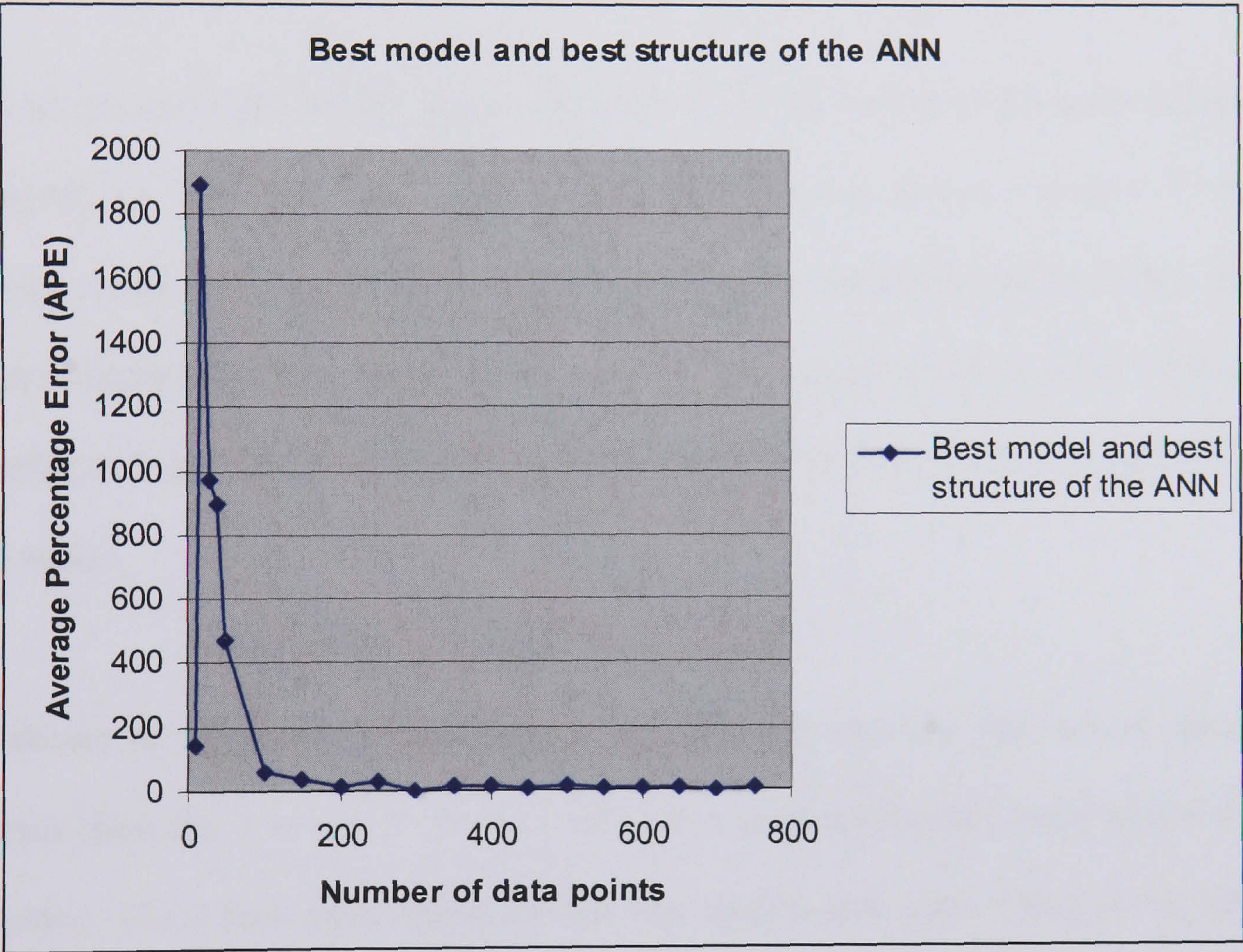


The best structure was identified from Table A5.13 and the best model was identified from Table A5.12, as shown below. Varying the number of data points meant the best model and best structure were effectively the same (2 fuzzy sets, 70 epochs).

**Best structure: (2 fuzzy sets, 70 epochs, 50 data points) (272.4(APE), 682.8(APSD))**

**Best model: (2 fuzzy sets 70 epochs 500 data points) (10.8(APE), 24.3(APSD))**

**Figure 5.4: Best Model and Best Structure of the ANFIS in Experimental Trials 7**





**5.10 Experimental Trials 8: Subtractive Clustering Based Method, Including Comparing Using Multiple Linear Regression (MLR) (from Experimental Trials 9)**

Table A5.15 shows the results of the individual trials using the subtractive clustering based algorithm. Again there are very poor results (2103% APE in model number 1) and very encouraging results (5% APE in model number 16). Eleven of the models were below 50% APE. The clustering algorithm gave some of the best results of all the methods used.

Table A5.16 shows the MAPE for the normalised cluster influence for each variable of the model, i.e. X1, X2, X3, and X4 input variables and output variable, Y. In all variables cluster influence C (0.5) and D (0.75) were comparably the best. The results for best structure and best model are shown for both Experimental runs for subtractive clustering in Table A5.17, as well as worst structures, worst models and overall APE for all 16 trials.

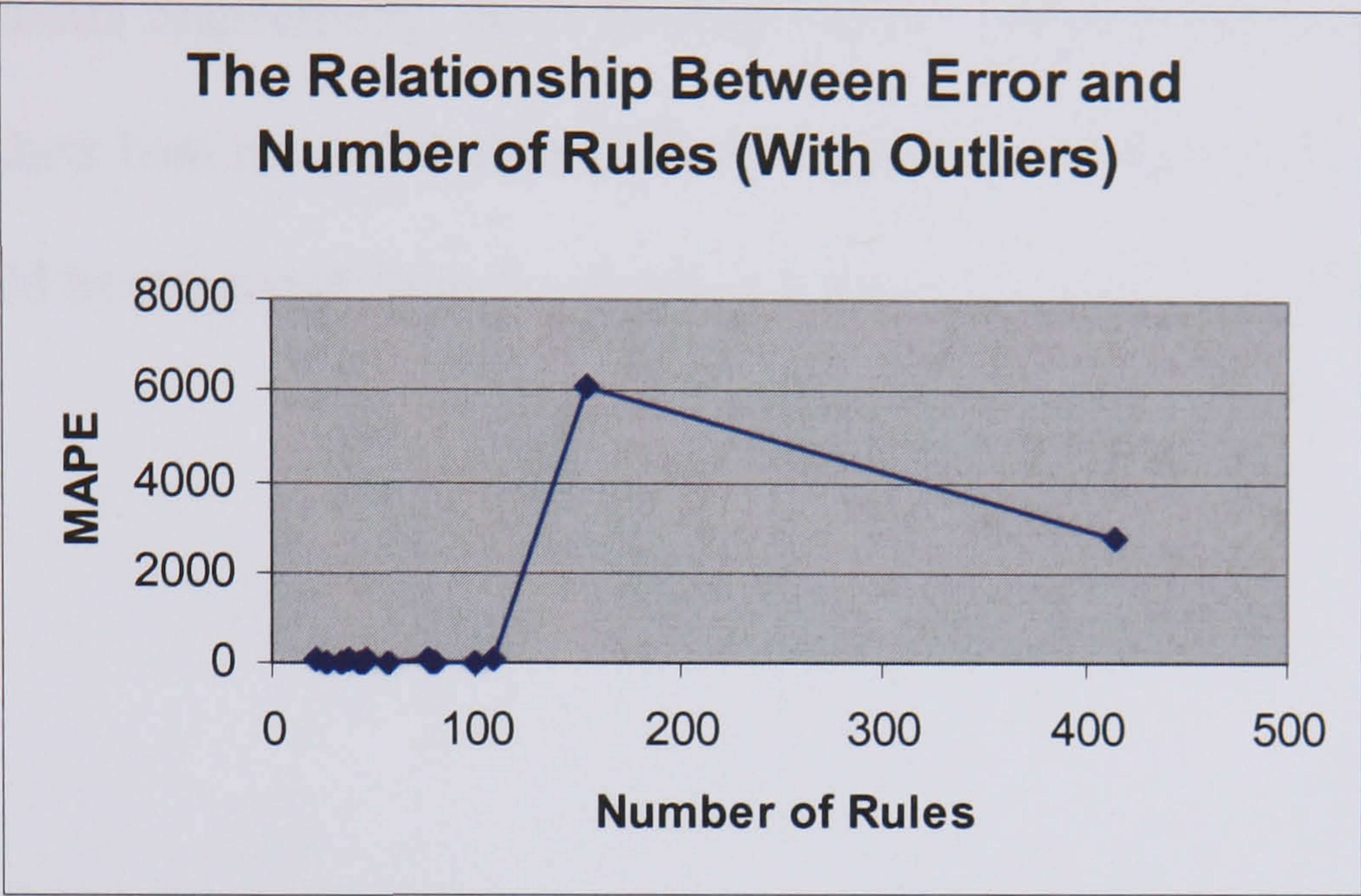
It is shown in Table A5.17 that there is variation between the best set of structural elements from run 1 to run 2. These 2 structures gave results that were within 6% of each other. The 2 runs gave results for the best models that were within 0.3% APE of each other.

Figures 5.5 and 5.6 show the relationship between the model's APE from Table A5.15 and the number of rules in the model. The number of rules in a clustering model is equal to the number of clusters, i.e. a cluster is split into an input and output to form a rule. It



is noticed that surprisingly there is *no* general trend relating an increasing number of rules in the model to an increasing APE.

**Figure 5.5: Number of Rules Versus MAPE From Clustering Run 2.**



**Figure 5.6: Number of Rules versus MAPE Without Outliers.**

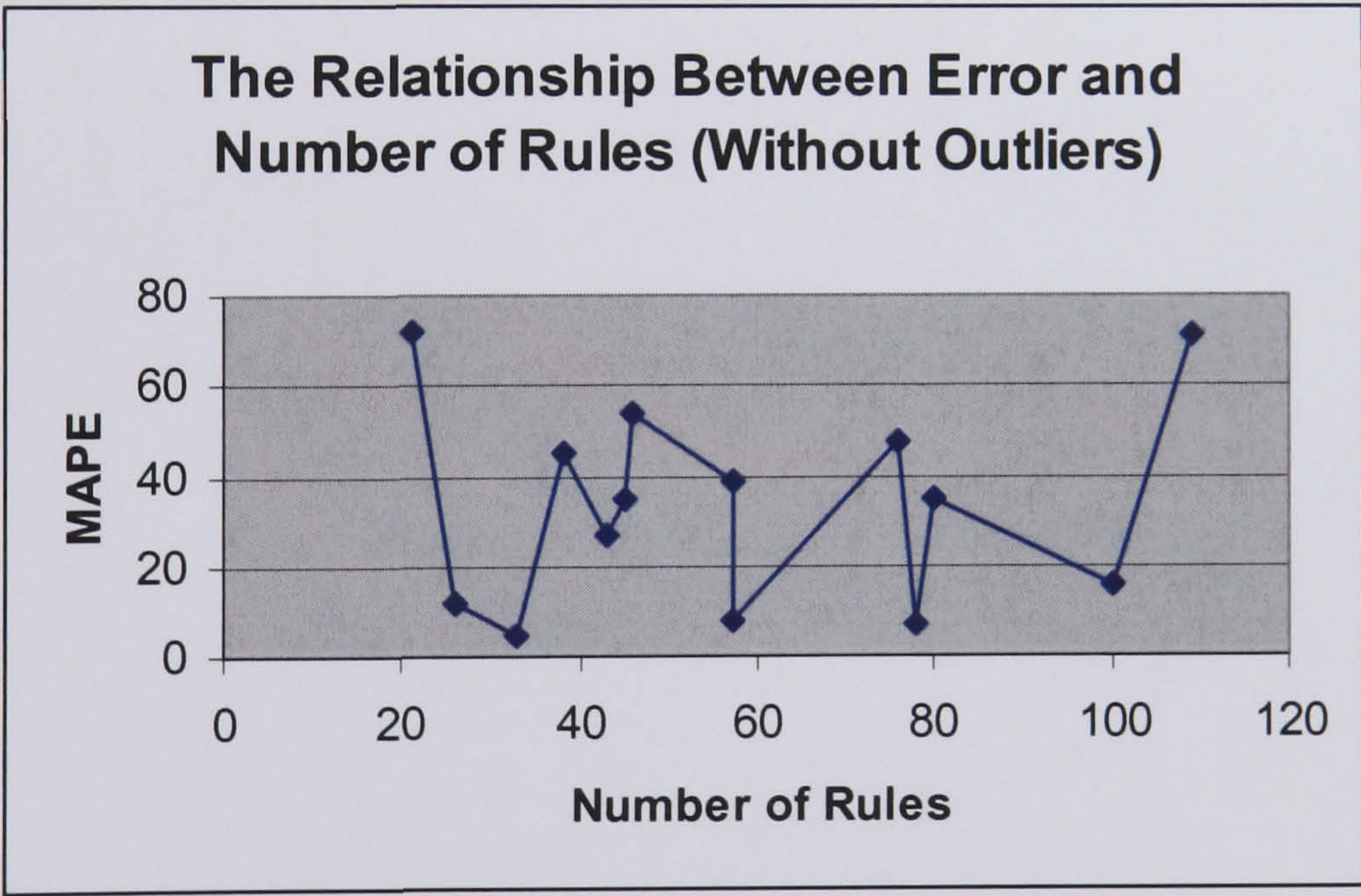


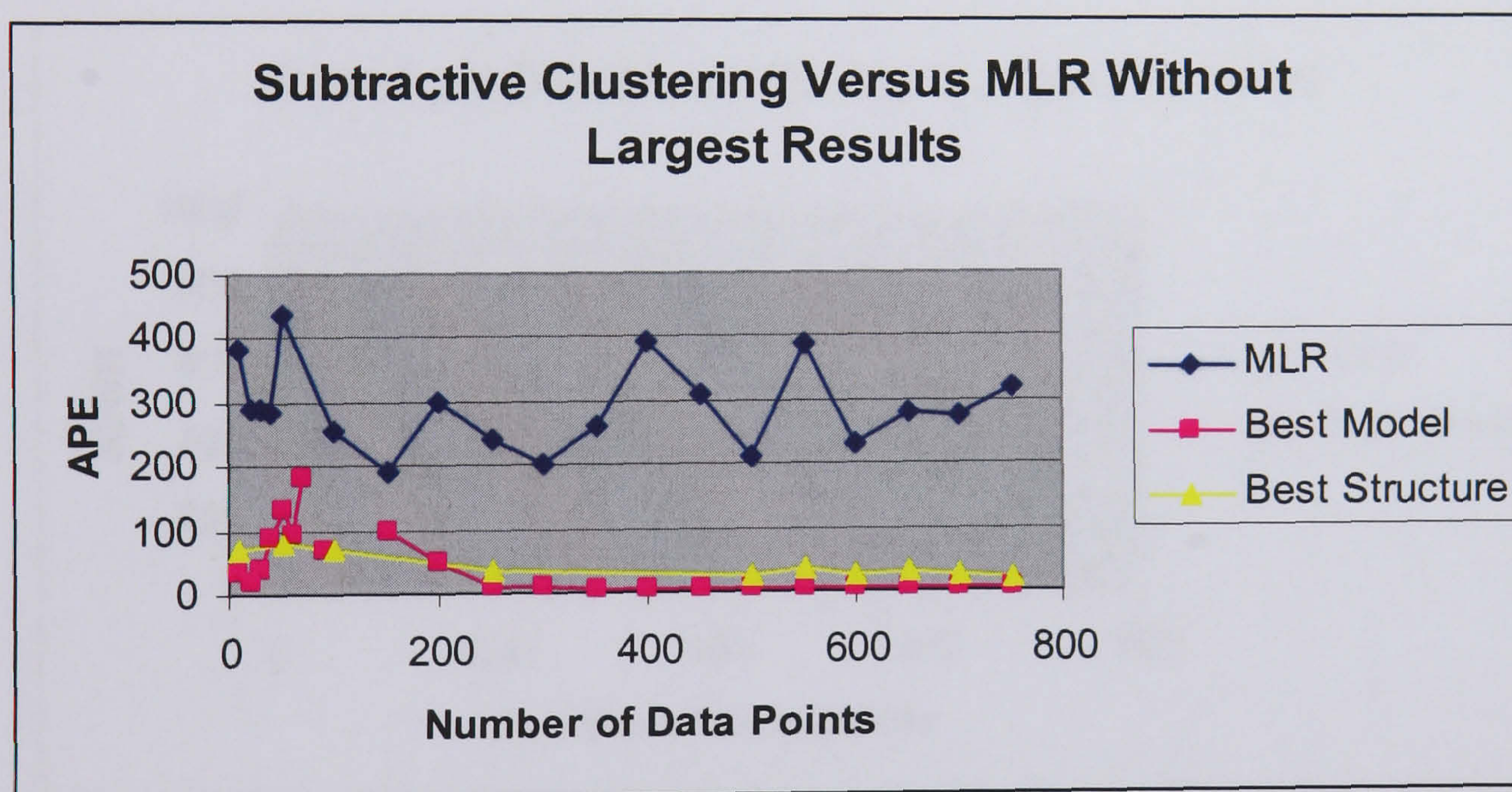
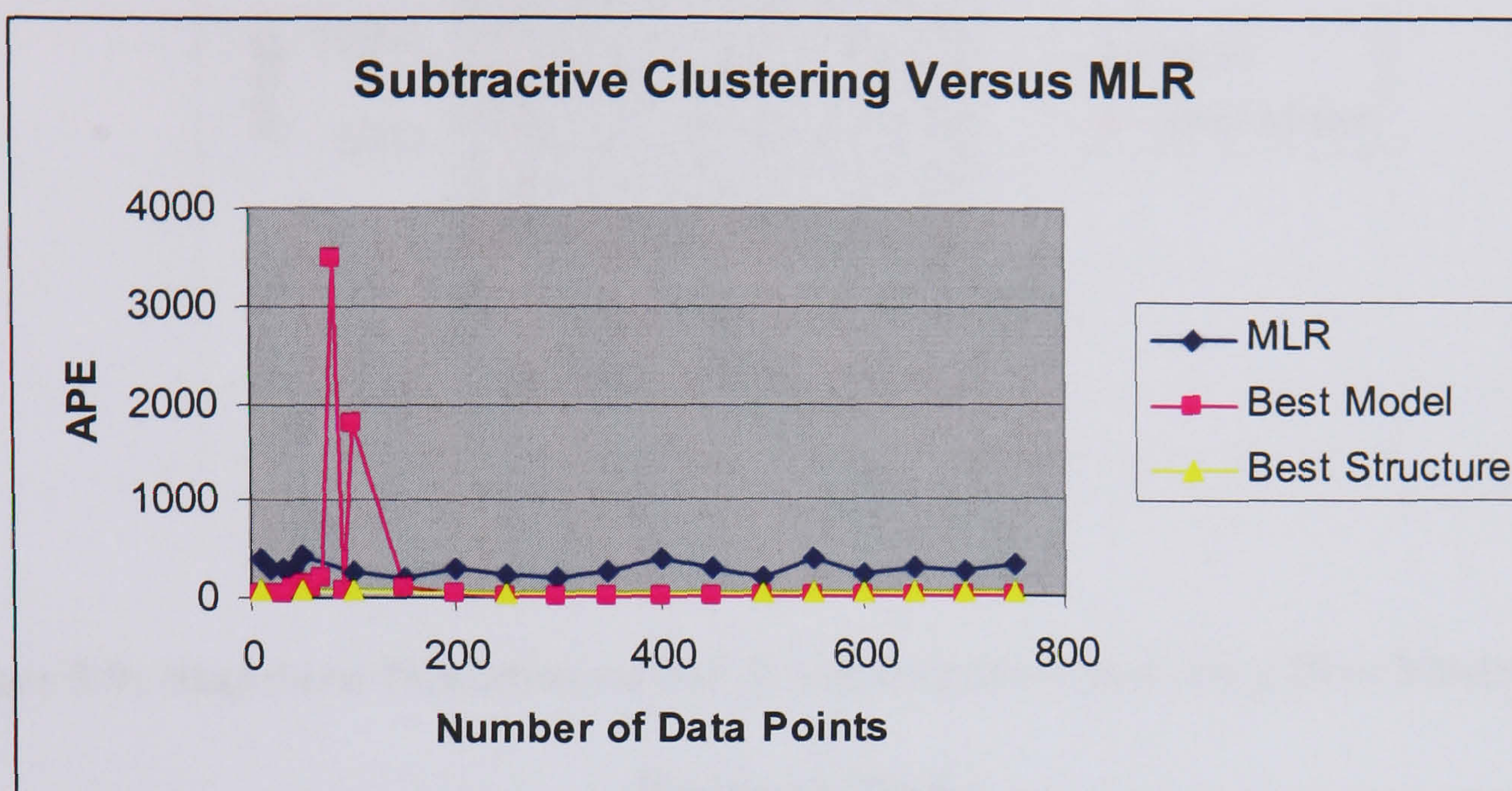
Figure 5.7 shows how the best model and best structure of the subtractive clustering based method and Multiple Linear Regression compared over a varying number of data



points. The subtractive clustering best model gave good results for a low number of data points, i.e. 32.0%, 21.8%, and 36.9% APE for 10, 20 and 30 data points respectively. But poor results are produced for a relatively low number of data points, e.g. 134.5% and 179.9% APE for 50 and 70 data points, and also 3492.4% and 1794.0% APE for 80 and 100 data points respectively. Both the best model and the best structure seem to “converge” to their best results at about 250 data points. It could not be predicted how many rules could be produced from the clustering algorithm and the normalised variable influences.

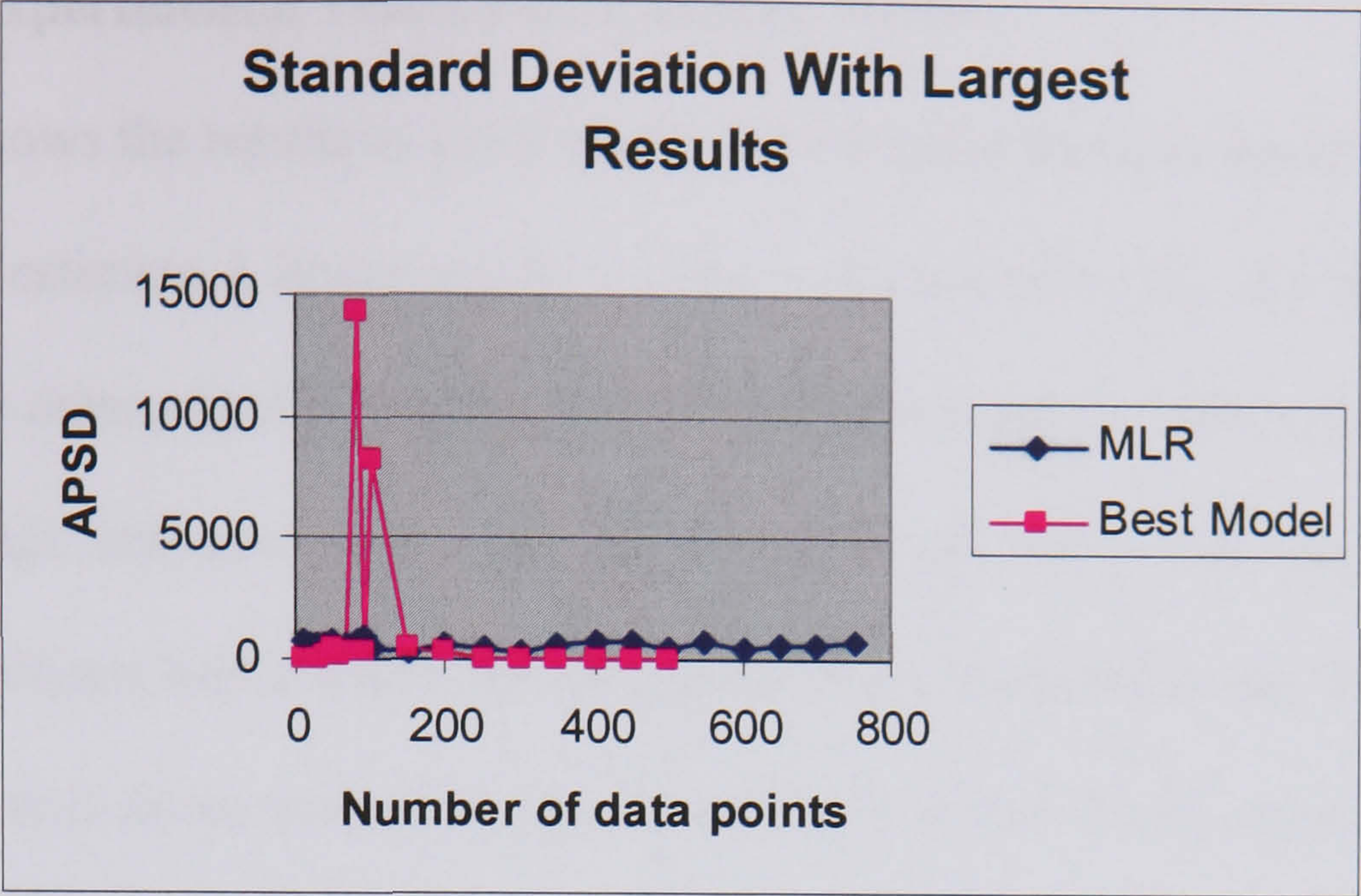


**Figure 5.7: Comparison of the APE of the Best Model and Best Structure of the Subtractive Clustering Method and Multiple Linear Regression for Varying Number of Data Points.**

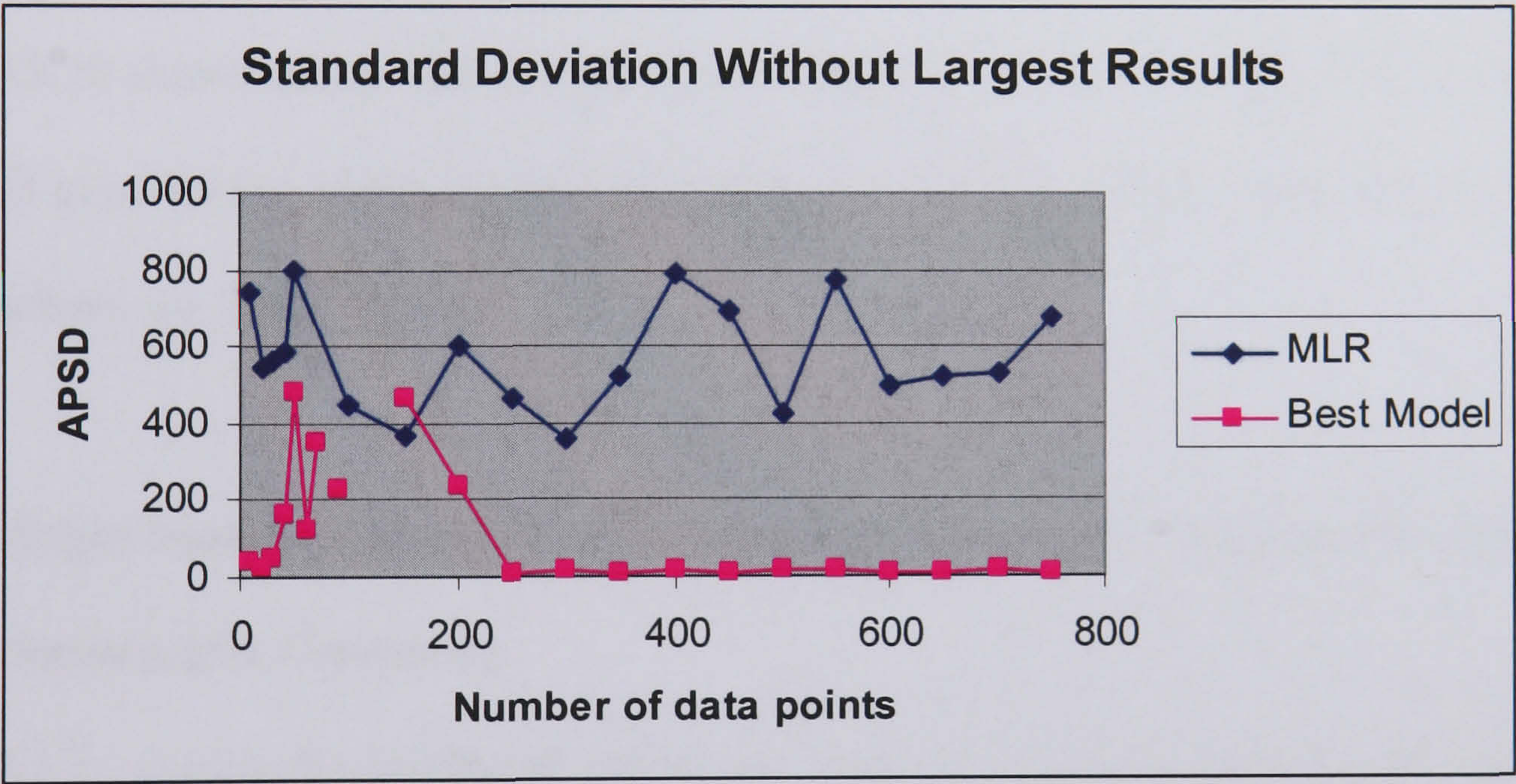




**Figure 5.8: Standard Deviation of MLR and the Subtractive Clustering Best Model**



**Figure 5.9: Standard Deviation of MLR Subtractive Clustering Best Model Over a Reduced Range**



As shown in Figures 5.8 and 5.9, the magnitude of the Average Percentage Standard Deviation follows a comparable trend for the APE within the subtractive clustering results for Equation (4.1).



**5.11 Experimental Trials 10: Testing the Effect of the Best Model Structure from Experimental Trials 4 on a Linear Model.**

Table A5.18 shows the results of individual trials of using the best model structure from Table A5.9 to estimate a linear model in four variables (X1, X2, X3 and X4) whose coefficients are determined by the Taguchi orthogonal array in Tables A4.7 and A5.19. The results range between 12.0% and 15.9% APE, and show that the best model in Table A5.9 performs better when estimating the linear models chosen by Table A5.19, rather than the non-linear model, i.e. Equation 4.1. The best model structure is adapted to the linear model by fitting it to the range of variables within the linear model and calculating the appropriate rules in the same manner as in the previous Experimental Trials 1-4.

Table A5.20 shows the results of applying the MLR method to Equation 4.3. It is shown the MLR predicts the linear model perfectly for the number of data points used, even for low numbers, i.e. 5 data points.

**5.12 Experimental Trials 11: Identifying Interactions in Experimental Trials 9, Subtractive Clustering.**

Table A5.21 shows the results of individual trials of the subtractive clustering based method, as in Table A5.15, but on this occasion a different orthogonal array has been used and only 2 levels of normalised cluster influence (0.2 and 0.75). The new array incorporates interactions between levels of normalised cluster influence as shown in Table A5.22. Table A5.21 has a “best model” of 4.4% APE.



Figure 5.10 shows graphs of interactions between normalised cluster influences of different variables. For example the graph  $X3 \times X4$  would expect 2 parallel lines if the cluster influences of  $X3$  did not interact with those of  $X4$ . The converging lines indicate a weak interaction. The graphs of  $X2 \times X4$  and  $X2 \times X3$  over different cluster influences show strong interactions as plotted lines nearly cross or cross respectively.

**Figure 5.10: Graphs to Show Interaction Effects of Cluster Influence for Each Input Variable**

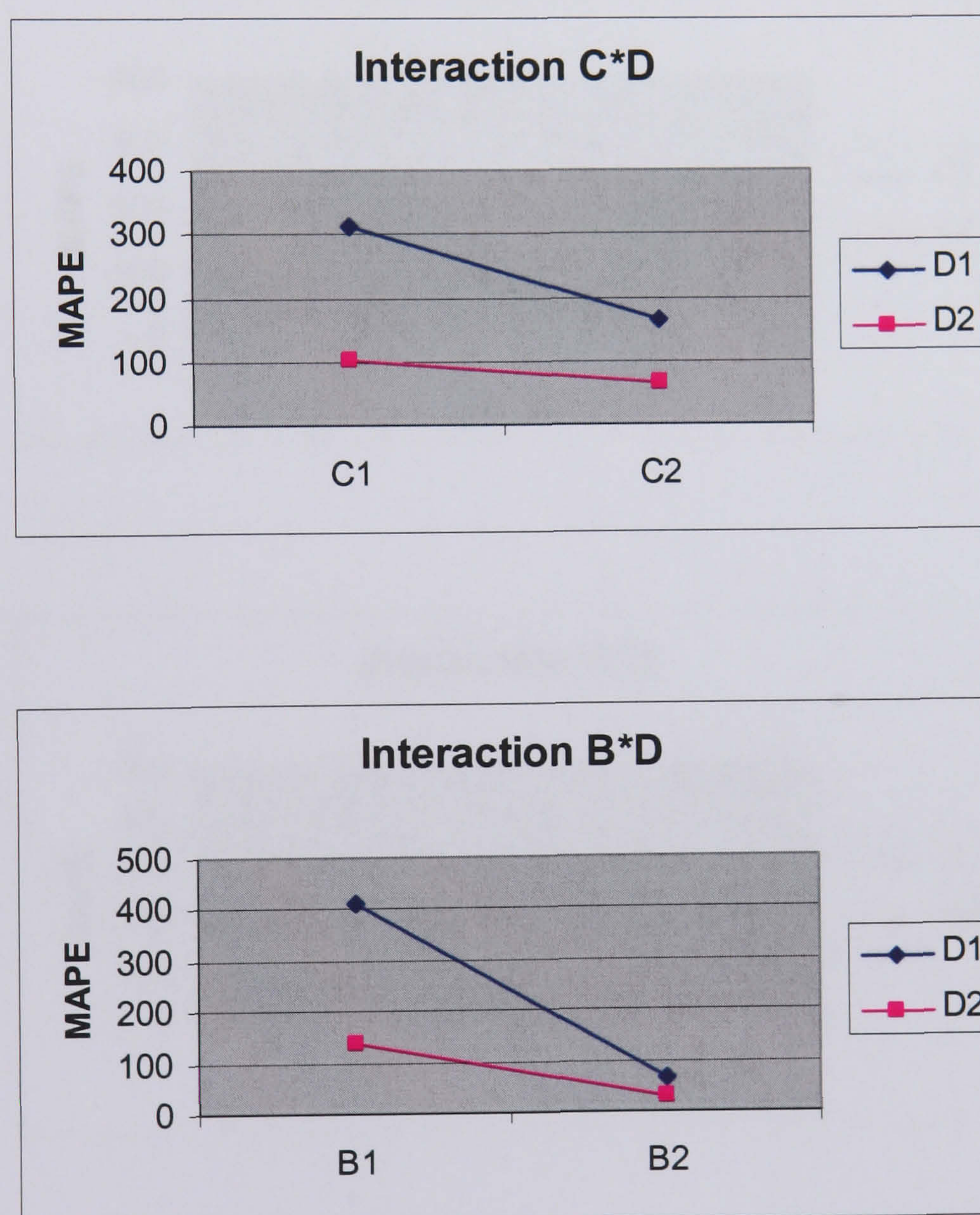




Figure 5.10: Continued

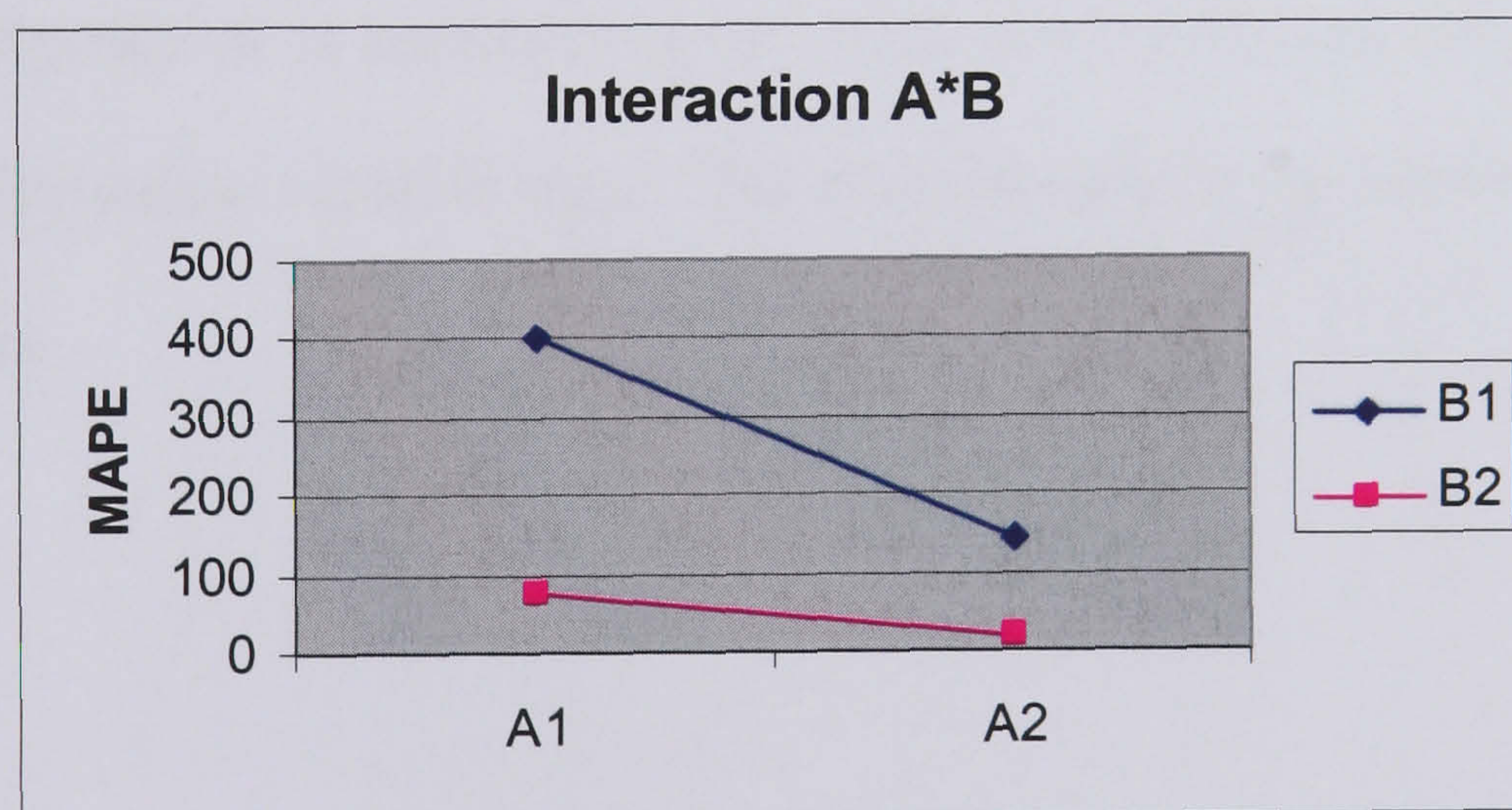
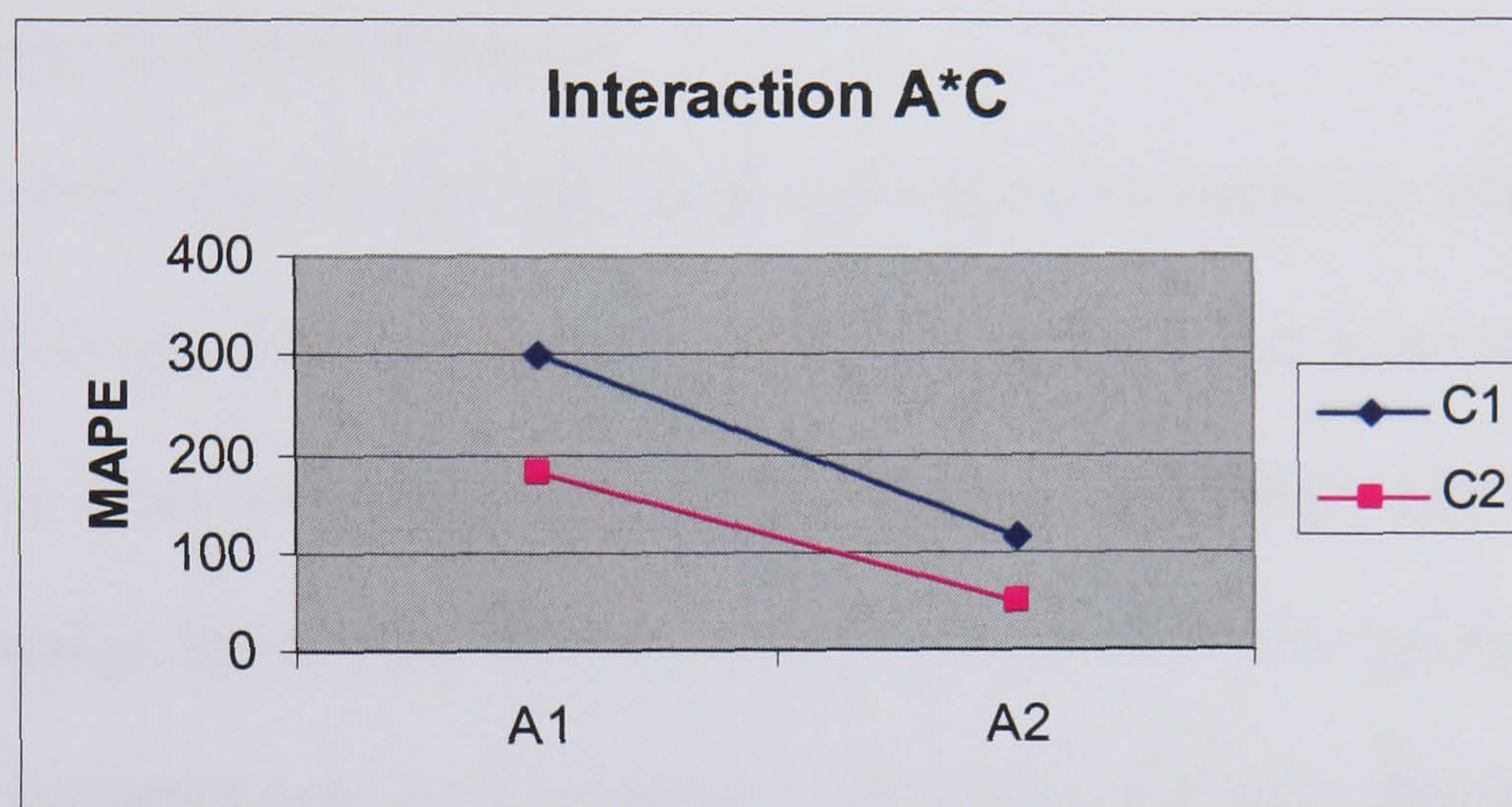
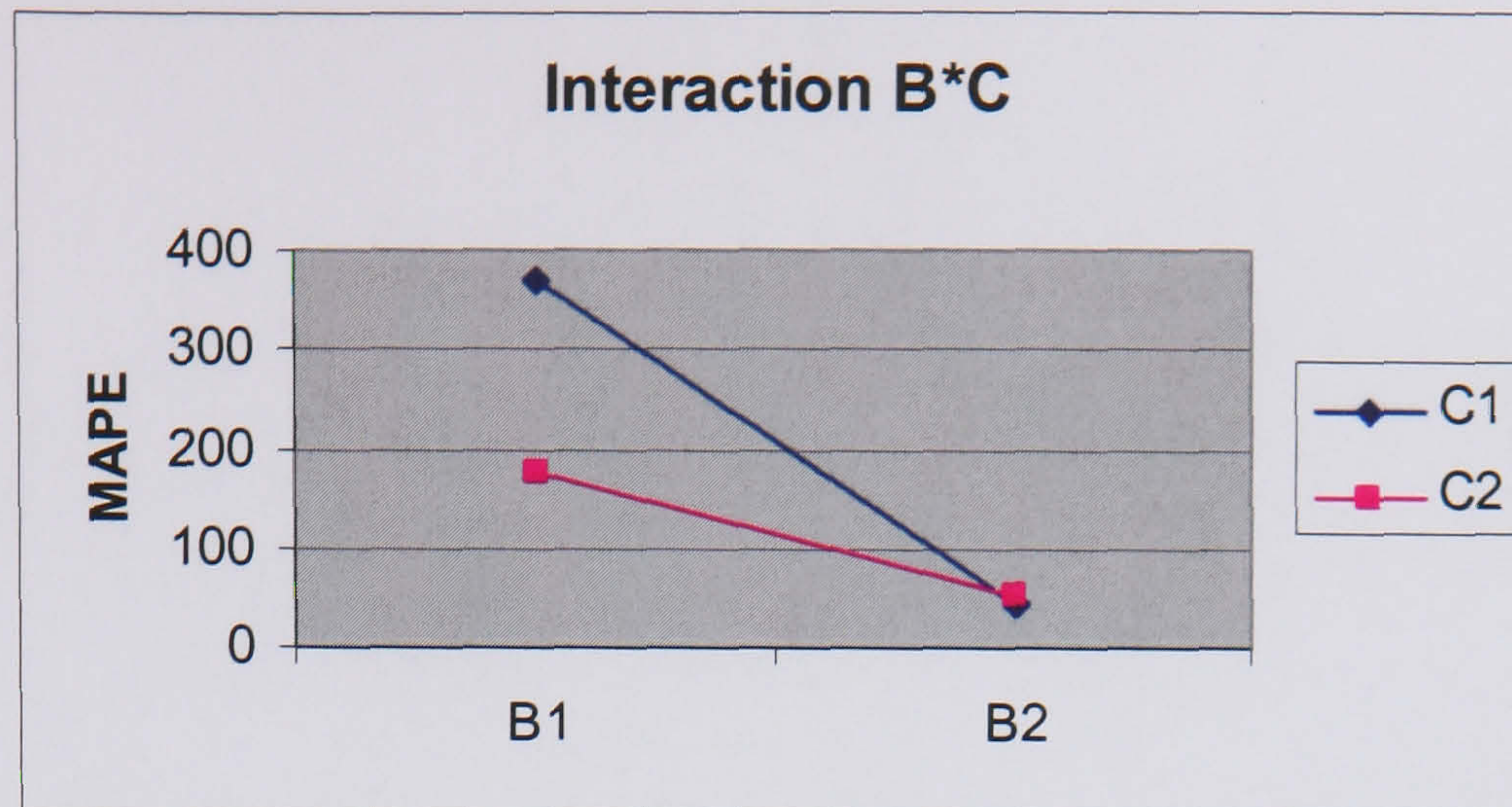
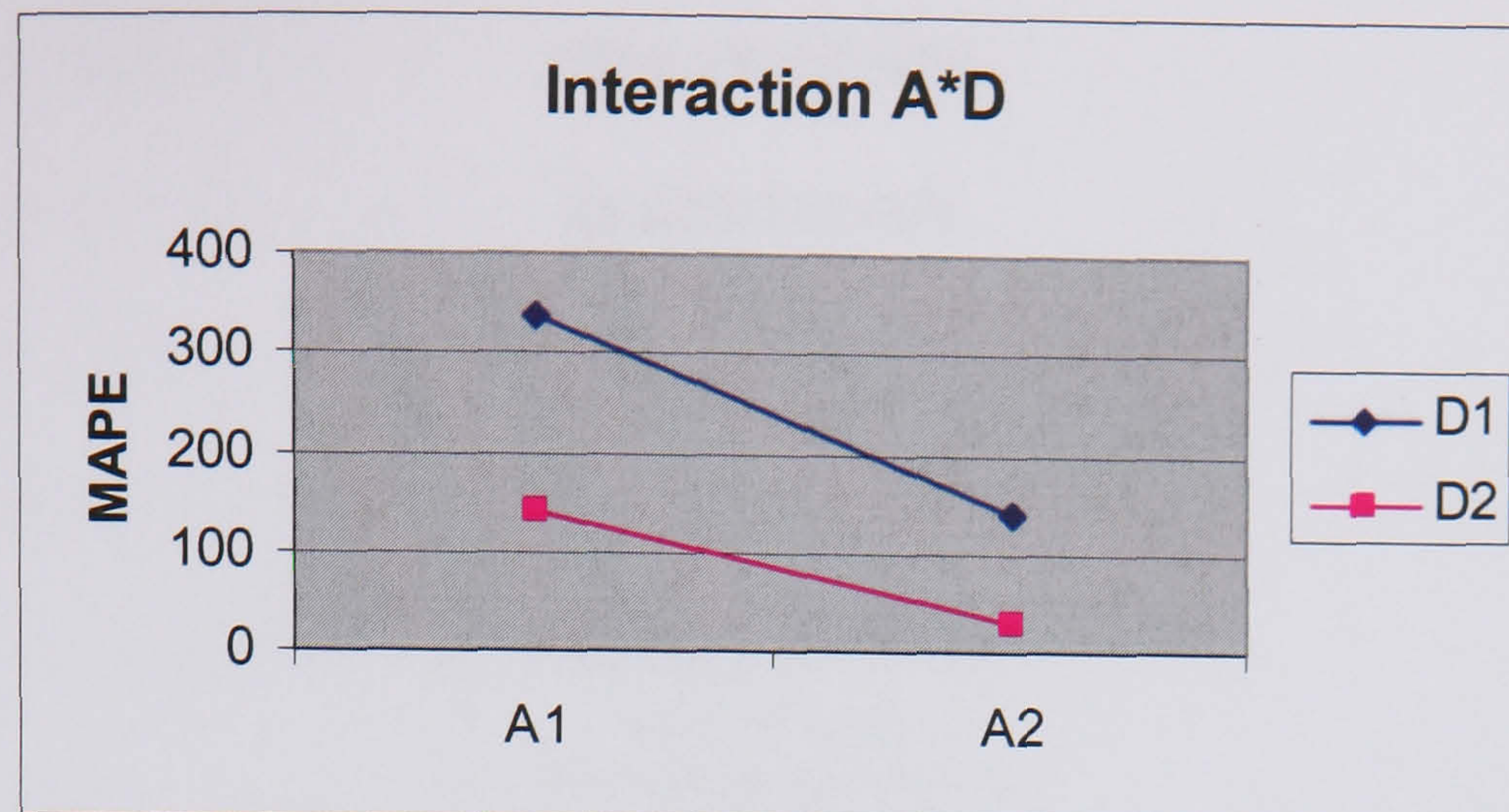




Figure 5.10: Continued



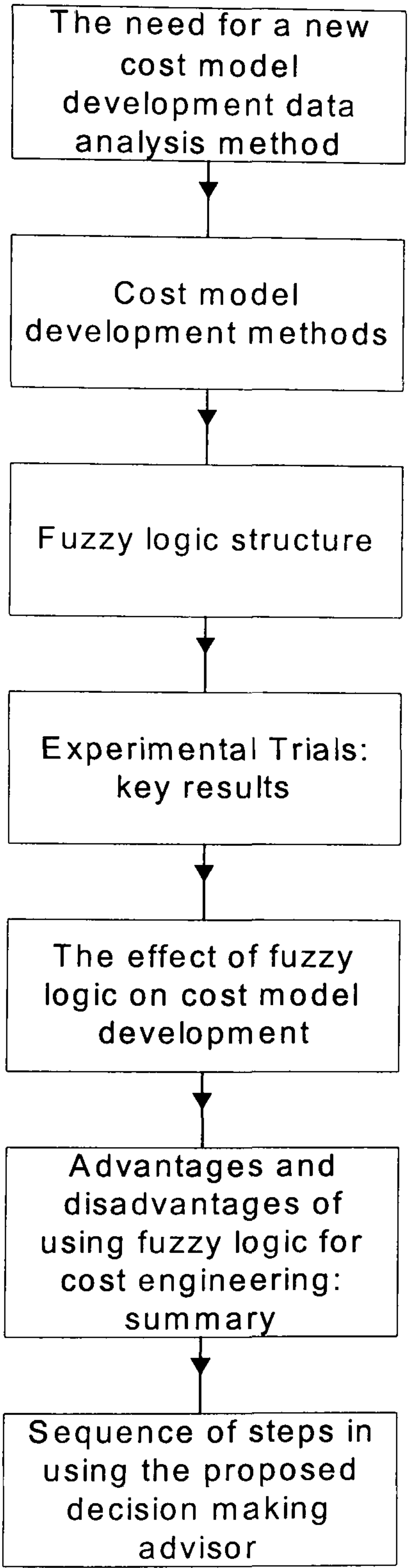
### 5.13 A Change in Error Measure

Mosher et al (1999) describe the General Error Regression Model in which APE is used. In this research it was found that small errors compared to an expected smaller value could produce a large percentage error of the order of thousands that in turn produced rather large Average Percentage Errors, i.e. when the mean of the percentage errors was calculated. Key experiments were repeated to determine Average Absolute Error (AAE) and Average Absolute Standard Deviation (AASD), as shown in Table 5.23, for comparison purposes. It is noticed that the AAE and AASD are also expressed as a percentage of the output variable range. The absolute error is the actual error expressed just as a number.



**CHAPTER SIX**

**DISCUSSION**





## **6.1 The Need for a New Cost Model Development Data Analysis Method**

### **6.1.1 Environment: the Need for a New Cost Modelling Method**

Decreasing time to market means streamlining, and making concurrent, existing New Product Development processes (Prasad 1996a, Prasad 1996b). Reducing data collection and analysis is one way of accomplishing decrease in “waste”, as perceived by lean engineering (Maskell and Baggaley 2004). Data reduction affects cost model development processes by innovation in methods that rely less on statistical significance, and hence less on large numbers of costly, and increasingly irrelevant data points. Fuzzy logic is advantageous as it can be both expert driven and data driven. This means that the efficacy of the method can be tested using data points as in Chapter 5, but not detract from its use with process experts. *The problem then is, can the process experts choose relevant fuzzy logic structural elements.*

Expert opinion can be used not just for data reduction. Contemporary manufacturing systems are more complex and hence hold a greater challenge for cost modelling. Fuzzy logic aims to overcome such complexity by utilising subjective perceptions of the behaviour of a complex system, much the same way as a process operator perceives the behaviour of complex forces during a hypothetical chemical process in order to control them. Therefore, by analogy, a small change in temperature (or weight) will cause a medium change in product yield (or cost).

Contemporary manufacturing has many sophisticated computerised applications, for example Enterprise Resource Planning Systems, and Computer Aided Design, to name only



a few (Maropoulos et al 2003, Maropoulos 2003). It is therefore important that innovative cost modelling methods should be able to operate with and through such systems. Fuzzy logic can be programmed through current tools such as “MATLAB fuzzy logic toolbox” and “Fuzzy Tech”. More importantly data held in database format can be processed through ANFIS or clustering algorithms to produce a Fuzzy Inference System. Such a FIS can be further modelled via experts, or interpreted. *It is emphasised, though, that further work needs to be done regarding the interpretation of data driven FISs and establishing knowledge about how expert driven and data driven models are related for the same application.*

## **6.2 Cost Model Development Methods**

### **6.2.1 Cost model development: why use fuzzy logic?**

Fuzzy logic applied to cost model development can be summarised as:

1. Transparency through linguistic variables and if then rules increases understanding to a large variety of personnel.
2. The leverage of expert opinion impacts a potentially large resource found in data collection.
3. Provides a consistent and effective framework in which to hang subjective opinion, skill, knowledge and experience of estimators.

It is immediately clear that there is a need for a transparent and easily understandable cost modelling method. Fuzzy logic’s potential to use linguistic variables and a set of rules distinguishes the method as a candidate for one whom is more readily understandable than



an esoteric equation with no clues as to how it might be used properly, or how it was derived in the first place.

The notion of concurrent engineering has meant the use of “Integrated Product Teams” for product development as the representation of one facet of inter-departmental integration. It is therefore apparent that cost modelling in such an environment will touch a large variety of personnel with different backgrounds and expertise. Particular problems are caused by using cost models in such environments, i.e. communication of costs, understanding the cost model, applying the cost model at the right level of detail, and understanding the evolution of the cost model development process in relation to the product development process.

## **6.3 Fuzzy Logic Structure**

### **6.3.1 Fuzzy Logic Structure**

*In order to develop a decision making methodology for using the fuzzy logic method a structure must be imposed.* There are 3 levels of structure proposed within fuzzy logic for studying cost model development.

- (1) Level 0: the method used to generate the general method, for example ANFIS and subtractive clustering produce a TSK method.
- (2) Level 1: the general method, for example the Mamdani method. Examples of the Level 1 methods are the fuzzy logic methods identified in Section 3.4
- (3) Level 2: the sub-methods of the general method, for example the product operator for AND (Appendix A1).



The structures used in the numerical experiments are shown in Appendix A1. Examples of Level 1 methods are the fuzzy logic methods in Section 3.4.2, 3.4.3, 3.4.4 and 3.4.5., i.e. Mamdani, TSK, ANFIS and subtractive clustering. It is therefore apparent that some methods, such as ANFIS and subtractive clustering are both Level 0 and Level 1 methods. This research has therefore clearly structured the fuzzy logic method.

In particular, the Fuzzy logic method is studied by the concerns of the:

1. general *process* of fuzzification, inference, defuzzification, and
2. use of *structural elements* of fuzzy sets (fuzzification), rules, AND and OR methods, implication methods, aggregation methods (inference), and defuzzification methods within the numerical experiments.

It is clear from the numerical experiments in Chapters 4 and 5, that the choice of each element of the structure as defined in the levels 0, 1 and 2, above, affects the estimating accuracy of the model. It is also clear that Level 0 is affected by Level 1 that is in turn affected by Level 3. There are some observations to be made about the structure that has been imposed:

- fuzzy logic structure is dependant on application, for example the Mamdani and Takagi Sugeno Kang methods have been used predominantly in the field of control systems and the characteristics of that application (Kickert and Mamdani 1978) and clustering methods have been used in Group Technology schemes (Gindy et al 1996),



- some parts of the structure are fixed dependant on the fuzzy logic method (Level 1), for example the weighted average defuzzification method is used for the Takagi Sugeno Kang method,
- when considering structure at the fuzzy set level (Level 2), it is noticed that fuzzy sets allow a theoretically infinite number of choices, i.e. their shape can be represented via an infinite number of potential functions. A significant few of these functions, pertaining to popular shapes such as triangular or trapezoidal, have been used whilst being justified by linguistic descriptions and through the concept of degrees of membership to a set,
- the potential for particular types of rule bases (considered in Section 3.3.2), as affected by Levels 1 and 2, to increase exponentially in number, with the addition of fuzzy sets and variables, is an important problem to solve, for example by the rule reduction methods in Section 3.3.2. Without resolution of this problem data collection problems can preclude use of the fuzzy logic method altogether (for example cost model 9 in Experimental Trial 1 had  $7^4$  (2401 rules) (Section 4.6.1), and
- universal function approximation proofs occur for different structures at all 3 levels. Universal function approximation means particular examples of FIS can approach perfect accuracy. It is clear that a theoretical proof that some methods are capable of any accuracy can be essential (Kosko 1994). A summary of universal function approximation proofs are shown in Appendix A1. *Universal function approximation proofs are not prescriptive, i.e. they do not give a method as to which fuzzy logic*



*structural elements are required to obtain a given accuracy. Hence there is clearly a need for a systematic method of choice of fuzzy logic structural elements.*

### **6.3.2 Systematic Methods**

Previous research applied to fuzzy logic and cost modelling has not been applied in the context of a *systematic* cost model development. In particular:

1. there is a need for general data sources from which structural elements have been derived, but have been too general, e.g. data sources named as “literature” or “experience”, to be equipped with systematic methods for supporting specific detail about “literature” or “experience”. For example the workflow required to convert literature sources into fuzzy sets,
  2. there have been no attempts at structuring development of fuzzy logic models via a consistent series of tasks, e.g. data identification, data collection or data analysis,
  3. no research to place fuzzy logic into a coherent cost model development process, and
  4. research has generated only one fuzzy logic cost model per application, instead of a systematically developed range of cost models in order to choose the best one.
- (Although Petley and Edwards (1995) explored the performance of triangular, flat and Zadeh’s curved membership functions in estimating the capital costs of chemical plants)

From Section 2.7 it is apparent that there is a gap in the cost model development process for modelling subjective uncertainty or experience in the proper manner. In addition, there is more scope and opportunity for the use of Artificial Intelligence methods in cost model



development, because of their recent development and variety of possible methods (Norvig and Russell 2003, Jang et al 1997).

Previous research, taken individually, has been insufficient for selection of fuzzy logic structural elements in a systematic process and for a range of applications or situations. There is a clear need to provide cost engineers a methodology for advice in choosing best practice fuzzy logic methods and structural elements. Such a methodology should be used for cost engineers for a range of experience with fuzzy logic. Such a methodology should also gather previous experience into one place.

The approach used in this research is to use the Taguchi methodology and a case base of previous research to inform a decision making methodology for the fuzzy logic method in cost model development. The approach is a valuable one because of its possibility to be improved in scope to analogous and wider application areas of fuzzy logic. *The need for a systematic method for fuzzy logic is a universal one (Chen and Chen 1998).*

### **6.3.3 Choice of Fuzzy Logic Methods**

Previous applications of fuzzy logic in cost or process time related applications as shown in Appendix A2, have not considered a systematic method for choosing fuzzy logic structural elements from among possibilities. Some research has chosen elements because of their track record of choice in other applications in different areas of manufacturing (Jahan-Shahi et al 1999). Some research has provided *isolated* additional reasons such as computational simplicity, or subjective notions that pertain to choosing fuzzy set triangular shapes or



others (Petrovic et al 1999). The problems regarding the current treatment of fuzzy logic and cost estimating can be summarised by:

1. the fact that structural elements have been chosen through popularity
2. the fact that structural elements from Level 2 have been chosen through the use of a pre-defined structure, e.g. through the Mamdani method (Zafiropoulos and Dialynas 2005)
3. the fact that structural elements have been chosen through the simplest structure, e.g. triangular fuzzy sets, low number of variables and therefore a low number of rules (Shehab and Abdalla 2001)
4. the fact that structural elements have been chosen through examples where choice has not been justified
5. there is no systematic method linked to accuracy, other than trial and error
6. there is no attempt to link structural elements to a general function (comparable to Equations 4.1 and 4.2) and hence accuracy
7. there is no direct comparison to regression found for the Mamdani, ANFIS or subtractive clustering methods
8. there is very little research to compare the different fuzzy logic methods identified in Section 3.6, in the same application. The only comparison example found was by Chen and Chang (2002) whom compared fuzzy regression and fuzzy goal regression with multiple linear regression analysis,
9. there is no research into a formal and structured method of obtaining process experts' facts and relating them to accuracy



10. there is no research to compare the use of different fuzzy sets, or defuzzification methods, and other fuzzy logic structural elements.
11. there is no research contemplating formal rule reduction methods, other than experience and trial and error
12. there is no research that contemplates the significance of the universal function approximation property
13. there is no examination into how one structural element affects another

#### **6.3.4 Experimental Results: Estimating Accuracy as Related to Structural Elements**

The principal discussion of the efficacy of fuzzy logic is through its estimating accuracy of all the cost modelling characteristics. The following discussion considered the experiments in Chapters 4 and 5. All discussion is in the context that: the results were a range of estimating accuracies for a range of fuzzy logic methods and for their *associated method of construction, in this case, and for the synthetic cost models in Equations 4.1, 4.2 and 4.3.*

The associated method of construction can be categorised as a Level 0 method as in Section

##### **6.3.1**

#### **6.3.5 Experimental Results: Initial Observations**

*Equations 4.1 and 4.2 were used in the numerical experiments in order to test the fuzzy logic method under conditions in which human error had been eradicated.* In particular resulting errors could be attributed consistently without variation due to subjectivity or inconsistency of human experts.



Initial observations were made with regards to the best cost models for each fuzzy logic method modelling data from Equation 4.1 and using two different error measures.

It is important to note that these results allow direct comparison of the efficacy of these methods under the conditions of:

1. the way models were constructed for these methods,
2. Equations 4.1 and 4.2 used to produce synthetic data.

Having said this it is also noted that general statements can also be formed about conditions the methods were tested under, i.e.:

1. one equation is non-linear, whilst the other is linear,
2. four input variables were used in these experiments,
3. certain numbers of data points were generated for the ANFIS and subtractive clustering based methods,
4. the data was accurate, unbiased and error free, and
5. the error measure used was the APE or General Error Regression Model (Mosher et al 1999), and the average absolute error in a limited number of cases (Table 5.23).

### **6.3.6 Systematic Method to Choose Fuzzy Logic Methods and Structural Elements:**

#### **Best Model and Best Structure Problem**

The need for a systematic method for choosing fuzzy logic structural elements and fuzzy logic methods has already been identified in Sections 6.3.2 and 6.3.3. Several approaches identified for this research are:



- (1) the Taguchi methodology and the use of numerical experiments and synthetic cost models as described in Chapter 4,
- (2) the construction of a Table of results (as shown in Appendix A2) from the literature, in order to add to and help validate the results reported in Chapter 5, and
- (3) an overall proposed decision making methodology.

The Taguchi methodology, in Chapter 4, allowed the choice of the set of structural elements that showed the least MAPE. For example in Table A4.1 and A5.1, those structural elements would be 7 fuzzy sets, 75% overlap, 0% shoulder width, 100% of the rule base and Mean of Maximum method . This was possible because of the implicit assumption, that there were no interactions between structural elements. It is noted that Experimental Trials 1, 2, 3, 4 and 8 allowed for a “best model” as well as a “best structure”. The presence of a “best model” *and* a “best structure” implied that some form of interaction was indeed taking place. In the absence of refining the existing experiments through including interactions, the recommended path by Peace (1993), is to adopt the new structure chosen by an existing “best model”, and to conduct more experiments including interactions, to further analyse the situation.

### **6.3.7 Trends Obtained from the Taguchi Methodology as a Systematic Method**

Because of possible interactions between elements, trends were not reliable from those experiments that did not consider them. In the face of this situation, it is possible to make an assumption that all interactions were weak, and hence large differences between levels of factors constitute a trend. The assumption that interactions are strong cannot be



considered as easily (trends obtained from the Taguchi methodology as a systematic method are shown in Section A6.1). Because the experiments showed that interactions were occurring through the existence of a best model and best structure, it is possible to assume that there is a strong possibility of interactions having occurred in the literature based models (Appendix A2) that were not explicitly considered or reported in their construction. It is also quite possible that interactions occurring between fuzzy logic structural elements are a significant aspect of the fuzzy logic method. Research into fuzzy logic operators has occurred through the subjective building of fuzzy logic structural elements (Yager 2003) in isolation from other structural elements. There was no research in the manufacturing application area found that explicitly considers the construction of fuzzy logic operators in the context of the many possible interactions with other existing fuzzy logic structural elements. For example there was no research found considering a new defuzzification method in the context of all the possible fuzzy set shapes used in variables. Further research should verify the role of interactions within fuzzy logic modelling within the application to manufacturing.

The problems with the “systematic method”, i.e. the occurrence of interactions are further illuminated in Section A6.2.

## **6.4 Experimental Trials: Key Results**

The key results and observations of the experimental trials can be summarised:



- Examples of the fuzzy sets from Experimental Trials 1 and 3 are shown in Figures 6.1, 6.2 and 6.3. Their main features are that they are:
  1. symmetrical,
  2. equally spaced,
  3. all the same shape per cost model, and
  4. each variable has the same number of fuzzy sets.
  
- Examples of the fuzzy sets from Experimental Trials 8 are shown in, Appendix A6, Figures A6.4, A6.5, and A6.6. Their main features are:
  1. symmetrical,
  2. various spacing,
  3. the same shape type per cost models, and
  4. each variable has a varying number of fuzzy sets.

Instances in which fuzzy sets are asymmetrical and unequally spaced, and in which there are a variety of shapes per cost model per variable are not considered in the Experimental Trials. It is apparent, therefore, that the cost models' errors have an appreciable scope for improvement (and indeed worsening). The numerical experiments provide a method for ranking fuzzy logic structural elements in terms of their ability to accurately model costs. But it is important to take into account interactions between structural elements in the same



cost model, so that the ranking is accurate. The fuzzy sets used in the numerical experiments can be interpreted in meaning as representing an imprecise number. An example of imprecision in this respect is about 200 (triangular, Gaussian) or approximately between 100 and 500 and certainly between 200 and 400 (trapezoidal). The triangular and Gaussian shapes introduce a different meaning of “about” for each shape inferring several levels of detail in meaning. These meanings are shown in Table 6.3 and discussed in Section 3.5. Measures of imprecision are shown in Figure 6.1 and can be used in order to quantify subjective opinion.



Figure 6.1: Measures of Imprecision.

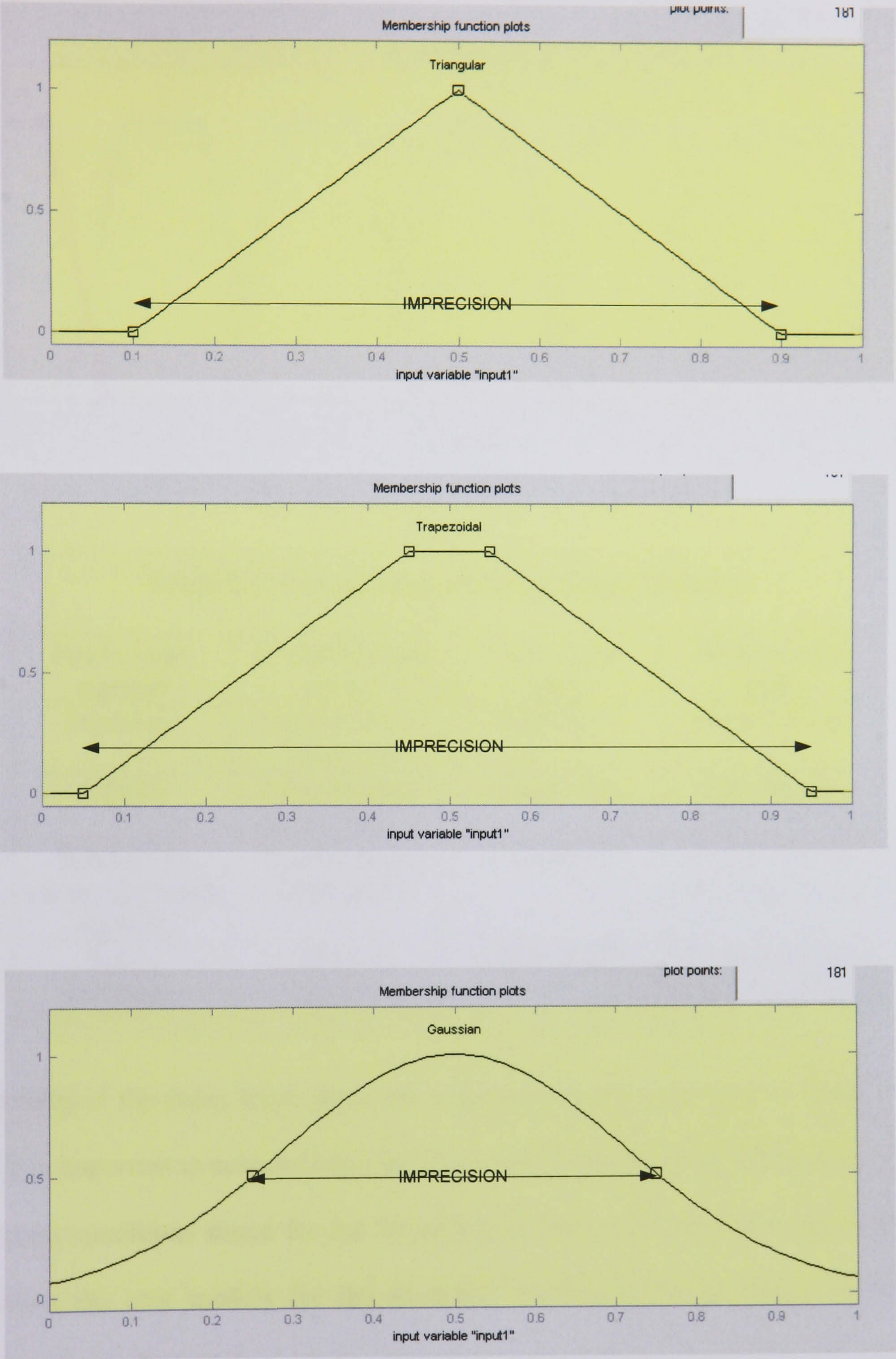




Figure 6.2: Cost Model 7, Experimental Trials 1

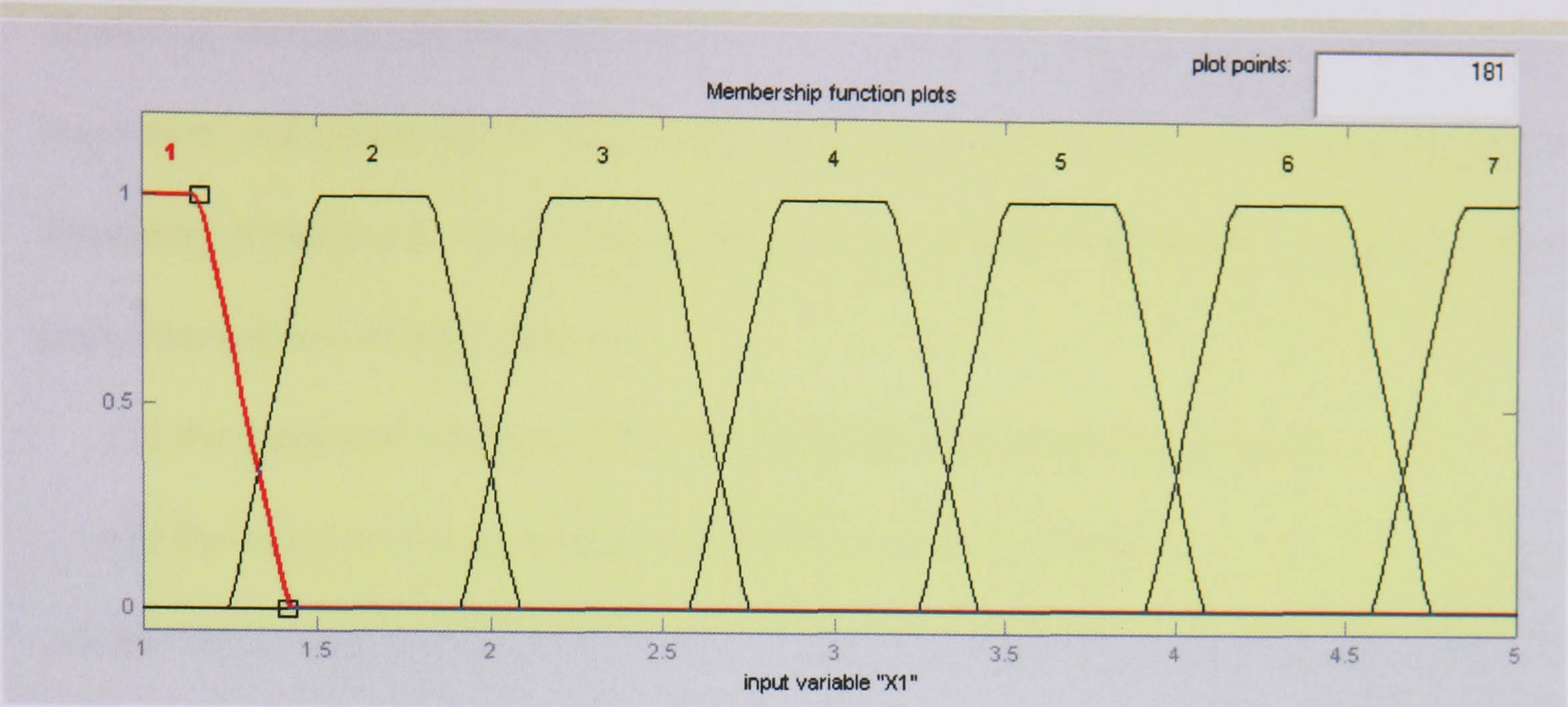


Table 6.1: Categorising of Fuzzy Logic Methods

Fuzzy logic method	Experimental Trial	Cost model type	Data source type
Mamdani	Experimental Trials 1-4	Parametric	Expert driven
ANFIS	Experimental Trials 7 and 8	Parametric	Data driven
Subtractive clustering based method	Experimental Trials 9,11	Parametric	Expert driven and Data driven
Fuzzy Arithmetic	N/A, i.e. literature	Grass Roots	Expert driven and Data driven

- A ranking of the fuzzy logic structural elements can be made for non-linear Equation 4.1. It is important to acknowledge that the ranking is objectively accurate only through the exact conditions stated for the Experimental design in Chapter 4. It is possible to construct the cost models for the Mamdani method in Experimental Trials 1-4 in different ways that might push the Mamdani method into top place above ANFIS and



clustering, in the rankings. A similar argument might be applied to the ANFIS and clustering methods. *In spite of this previous discussion, the results in Chapter 5 give important indicators as to how fuzzy logic structural elements will behave in the Mamdani framework, when fuzzy sets are to represent imprecise quantities.* Some important aspects to note, then is:

- (1) the ranges of values used for the variables in the experiments, and
  - (2) the amount of imprecision in the fuzzy sets per variables.
- All the linear cost models built using a Taguchi orthogonal array were estimated to a comparable accuracy by the best model structure from Experimental Trials 4. *This indicated that changing the slope of the linear model had little effect on error. Therefore the results might be considered on a more general level in that all the linear models had comparable errors. Further work should examine this possibility.*
  - A change of error measure was made for a selected few experiments. Significantly, it was found that absolute errors were perceived as improvements upon corresponding APEs in the few cases considered (Table A5.23). These results show how robust the model can be even when imprecision, as defined in Figure 4.6 and Figure 6.1, is large. For example in the “best model” in Experimental Trials 4, the imprecision of 75% standard deviation as expressed as a percentage of the variable range, i.e. 3 in X1, 675 in X2, 0.6 in X3, 74,250 in X4 and 275,008 in Y, made for a result of 15,098 AAE. Therefore, in comparison to large APE results, the relatively small AAE results indicated that the APE results were distorting them in some way. Examining the individual errors of each test data point one by one, found that small errors occurring at



the small end of the output range were being reported as hundreds or even thousands of percent error making the APE errors behave accordingly. With these considerations it must be strongly stated that APE is an accepted conventional error measure that takes into account relative differences in errors using percentages (Mosher et al 1999, Wang and Stockton 2001). Choice of error measure, therefore, has an effect on the decision making methodology.

- The numerical results can be summarised as a case base as shown in Table 6.2. This case base contributes to a proposed decision making methodology as shown in Section 6.7.
- Key observations of the numerical results are shown in A6.3

**Table 6.2: A Summary of the Numerical Experimental Results Through Ranking**

<b>Fuzzy logic methods</b>	<b>Rank ordering of fuzzy logic structural elements (from worst to best)</b>
Mamdani (non-linear Equation 4.1)	3,5,7 fuzzy sets
	25,50,75 overlap
	40,0,75 shoulder width
	25,50,100 percentage rules
	Min/min, min/prod, prod/prod AND/Implication
	Trapezoidal or Triangular, Gaussian fuzzy sets
	Bisector, Centroid, Largest of Maximum, Mean Of Maximum, Smallest Of Maximum defuzzification methods
ANFIS	50,300,500,750 data points, 2,3,4 fuzzy sets, 70,60,80,50 epochs
Clustering	0.75 is better than 0.2 for the input variables and 0.2 was better than 0.75 for the output variable, ignoring interactions



### 6.4.1 Deriving Meaning from the Models and Related Problems

A possible advantage surmised from the literature was the opportunity to derive meaning from the models produced, by a range of users. Meaning occurs through (1) linguistic variables, (2) “if then” rules, and (3) fuzzy set shapes. Fuzzy set shapes require an understanding of the concept of the degree of membership to a fuzzy set, but linguistic variables and rules use application specific knowledge in the form of words. They hence form an easy avenue of understanding the model directly. Cost engineers require that data sets be transformed into relationships between words describing cost and cost drivers for use in decision making, especially at tactical and strategic levels. Experimental trials 1, 2, 3, and 4 tested the use of fuzzy sets whose meaning can be derived from their place on the range of the variable. For example a Gaussian fuzzy set centred on  $X_1=5$  would constitute “about 5”. Alternatively this set could be termed “high”, for the linguistic variable  $X_1$ , whose other terms might be “medium” and “low”, if there were 3 fuzzy sets in total.

Table 6.3 shows meanings attributed to the fuzzy sets used in Experimental Trials 1, 2, 3 and 4. The meanings are based on the author’s intuition and knowledge of fuzzy set theory. These meanings are confirmed and added to through research shown in Section 3.6. It is difficult to find alternative meanings easily. The associated cost engineering issues are shown in Table 6.4.



**Table 6.3: The Meaning of Fuzzy Sets in Experimental Trials 1-4.**

Fuzzy Set	Meaning
Trapezoid (1,4,5,8)	Certainly between 4 and 5, and approximately between 1 and 8.
Symmetric Triangle (1, 4.5, 8)	Definitely 4.5 and approximately between 1 and 8. Equally possible to be between 1 and 4.5, as 4.5 and 8.
Gaussian (mean=4.5, standard deviation)	Approximately 4.5, the possibility of values between 4.5 and others fluctuate with the curve of the distribution. Gaussian distributions have other advantages other than meaning in terms of smoothness and their part in universal function approximation theorems.

The most potentially useful situation was the opportunity to derive meaning from the models that were generated by the data driven methods of Adaptive-Neuro-Fuzzy Inference System and the subtractive clustering based method. *A vital task of innovative cost modelling methods is to derive meaning about costs from large amounts of data, that otherwise is beyond easy understanding. Such meaning might be the identification of cost drivers, and also how changes in these drivers affect cost.* The term applied to such a task is machine learning. The reason for such importance is the huge amounts of data generated by large complex organisations in the computer age that can be easily processed using software.

The fuzzy sets produced by the subtractive clustering method are shown in Appendix A6, Figures A6.3, A6.4, A6.5, and A6.6. The algorithm produces fuzzy sets that vary in the manner as affected by the cluster influence (Chiu 1994). An initial question when considering the meaning of fuzzy logic models is how complete is the model? Adding a



rule changes the model's structure and output. Hence how many rules should there be? And does it matter if some rules attract more attention than others by having weights attached to them? Further work should address these issues.

Apart from the fact that the data driven methods gave good results in Chapter 5, this research contends that automatic systems identification, such as neuro-fuzzy modelling and subtractive clustering, presents some interesting options for applying meaning to large data sets of cost and cost drivers. A possible disadvantage, is that some research presents problems with interpretation (Lo 2003). For example, it is important to note that the number of rules interacts with the fuzzy sets within the subtractive clustering method to present problems of deriving an overall intuitive meaning of the model.

#### **6.4.2 Systematic Methods: a Closer Look at the Structural Elements used in the Numerical Experiments**

Once compared to the measure of imprecision as shown in Figure 4.6 and Figure 4.7, it is noticed that the output fuzzy sets for the best model of Experimental Trials 4, as shown in Figure 6.3, have an imprecision that is more than the range of the output variable. This fact exemplifies the robustness of the fuzzy logic modelling method, in that, a large amount of imprecision, as measured in Section 4.6.5, can result in good results as shown in Table A5.23. *It is the tolerance for imprecision and the robustness of the fuzzy logic modelling method that are a main strength.*



**Table 6.4: A Summary of Observations Linking the Fuzzy Sets and their Structural Elements to Cost Engineering Issues.**

Fuzzy set structural elements	Cost engineering issues
Type	Amount of imprecision in values, amount of subjective uncertainty about values, the meaning attributed to linguistic variables, data collection methods
Number	Amount of data collection, imprecision in values, resources needed to collect data
Overlap	Completeness of expert knowledge
Shoulder width	Amount of certainty attributed to ranges of values

### 6.4.3 Regression Versus Fuzzy Logic Models

Multiple linear regression analysis is a popular method within cost model development (Roy et al 2001). The issues with MLR borne out by Experimental Trials 9 are:

- Data points cost resources in collection and storage. Although this is the same for data driven fuzzy logic methods, the tolerance for imprecision of expert driven methods does not need this type of resource.
- It is assumed that data points exist. New technology does not have an effective and suitable cost modelling data analysis method.
- MLR is a great simplification of the relationship between cost and independent variable leading to many areas of over estimation and under estimation. A relationship between over and under estimates is shown in the Freiman curve (Asiedu and Gu (1998).
- MLR assumes a previous cost model form (e.g. linear or quadratic).

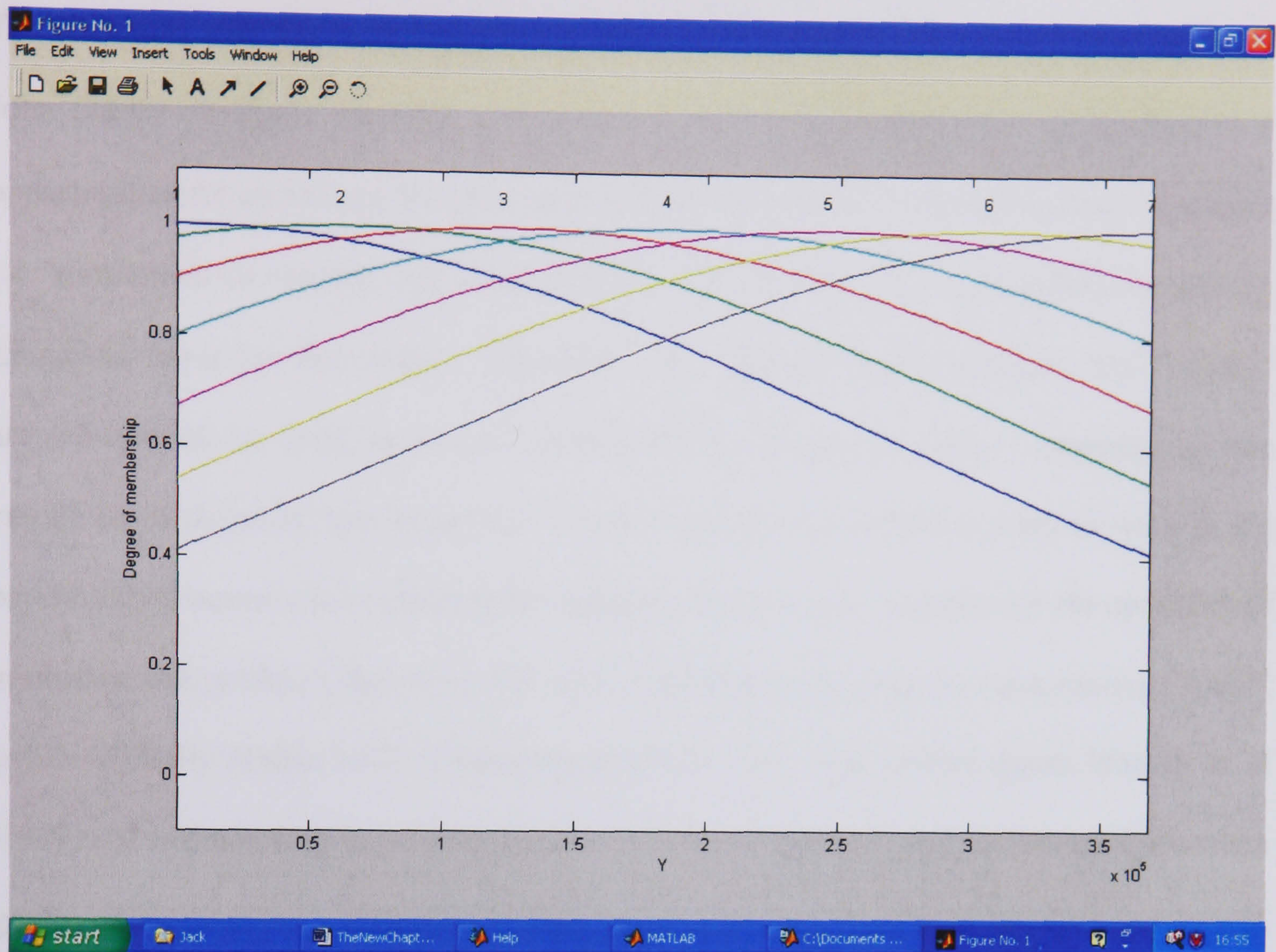


#### 6.4.4 Non-linear and Linear models

The *structure* of the best model from Experimental Trials 1-4 (that was used to estimate a non-linear function), used to estimate a linear function, as in Experimental Trials 10, produced better results in estimating a linear function when compared to the non-linear function. The 11.9% APE was achieved despite the very large imprecision, as shown in Figure 6.3 of the output fuzzy sets, i.e. more than the considered range of the output variable. This again indicates the potential for robustness of fuzzy logic models and their “tolerance for imprecision” as coined by Zadeh (Zadeh 1996). These comments are also appreciable when considering existing successful linear cost models (Rush and Roy 2001a, Kelly et al 1995), i.e. the success of fuzzy logic in estimating linear functions from imprecise quantities is highly relevant.



**Figure 6.3: Example Output Fuzzy Sets: Cost model 9 Experimental Trials 4**



#### 6.4.5 Deriving Meaning of the Subtractive Clustering Models

*The problem of choosing fuzzy logic structural elements is a fundamental one summarised in the statement: “The search for the best if-part sets in fuzzy function approximation has just begun” (Mitaim and Kosko 2001).*

Section A6.11 considers the meaning of subtractive clustering models. It is concluded that a cost engineer is potentially confused by the array of fuzzy sets and rules in this case and no clear advantage is gained. Section 4.6.8 and 4.6.11 shows insight through numerical results into how cluster influence affects model accuracy.



The literature also has something to say about the meaning of fuzzy sets. Garibaldi and John (2003) attributed meaning to non-normal and multi-modal fuzzy sets, so that new opportunities for attributing meaning in fuzzy logic based decision making were identified, i.e. restrictions to normal and convex fuzzy sets also restricted modelling capability. Linguistic terms for the variable “goodness”, i.e., “good”, “bad”, “normal”, and “saintly” are sub-normal, i.e. their degrees of membership are always less than 1, meaning no-one can be certainly good, bad, normal or saintly. Nguyen et al (1996) pointed to work in the psychology literature that examined the meaning of fuzzy sets. In particular the operators of minimum and product (Appendix A1) were identified as the best for representing “AND” psychologically (Oden 1977, Zimmermann 1978). Gill et al (1994) quote Watson et al (1979): “it is not in keeping with the spirit of fuzzy set approach to be concerned about the grades of membership”. Therefore joining the dots between a range of absolute certainty of truth and an absolute certainty of falsity, is used to create trapezoidal membership functions.

#### **6.4.6 Observations of the Numerical Results**

Key observations of results are found in Sections A6.3, A6.10, and A6.11. It is clear that direct comparisons between results are dangerous because of the difference between model structures and hence also the interactions that occur within them. Observation (4), in Section A6.3, leads to possible trade-offs between set-up costs and accuracy quantified by cost per rule formed. Cost per rule can be directly linked to the cost of an expert’s time if the model is to be expert driven.



#### 6.4.7 Standard Deviation

A low Average Percentage Error and a low Average Percentage Standard Deviation is the ideal situation for a model, i.e. low error and low variation in error. The numerical results in Chapter 5 and Appendix A5, provided standard deviations for best structures and best models in Experimental Trials 1-4, Experimental Trials involving the Adaptive Neuro Fuzzy Inference System, the Subtractive Clustering based method, Multiple Linear Regression Analysis, Experiments with a linear model, and testing for interactions. The research took the approach of focussing on the error measures of APE and AAE. Subsequently low error models were inspected for standard deviation results, i.e. to identify low error models with low variation in error. Accuracy is just one of the cost model characteristics. The cost model characteristics are linked to the experiments in Section A6.4 and reviewed in the context of fuzzy logic in Sections A6.5 to A6.7.

#### 6.4.8 Error Measures

It was found from Experimental Trials 1 to 4 that their method of construction was insufficient in conjunction with the examples of other fuzzy logic structural elements (e.g. defuzzification by centroid) to produce an “acceptable” APE *using the GERM error measure* (Mosher et al 1999). Further investigation found that the results in Experimental Trials 1 to 4 were “acceptable” if not “very encouraging” when the error was measured as an absolute average error and compared to the range of the output variable Y (as shown in Table A5.23).



In summary, for the Experimental Trials 1-4, method of construction was: (1) simply placing symmetric fuzzy sets with a certainty of one at  $X$ , to represent an approximate value, “about  $X$ ” or “between some values  $a$  and  $b$ ”; and (2) linking these sets via rules, as calculated from those exact values represented as data points (that indeed form the centres of these symmetric sets in a particular dimension). Exact values from equations 4.1 and 4.2 were used instead of expert opinion to eradicate human error from the results. Appropriate questions to ask cost engineers are those for determining imprecise values for linguistic terms and variables and the rules that connect them together and cost.

The “best structure” in Experimental Trials 1 produced an average absolute error of 15,098, where this worked out at 4.12% of the range of  $Y$ . This can be placed in the context of the range of  $Y$  being from 1020 (i.e. 15,098 is 1480% of 1020) to 367697 (i.e. 15,098 is 4.11% of 367697). This result places the method in a favourable light in some respects. The standard deviation was 13,086. Again this can be placed in the context of  $Y$ .

Further work will aim to construct a method of choosing more sophisticated fuzzy set shapes to further improve modelling error, and gain greater insight into the meaning of models.

Therefore use of error measure has an impact on the assessment of how well the method performs.



## **6.5 The Effect of Fuzzy Logic on Cost Model Development**

### **6.5.1 Range of Application Areas**

Fuzzy logic can be applied to any cost model form. Fuzzy logic can also be applied without having to know the cost model form (e.g. linear or quadratic). The best model, cost model 9, from Experimental Trials 4, when tested on a linear model, produced better results than for the non-linear model. The difference between the “best” APE of the linear models in Table A5.19 and that of the best model for the non-linear equation 4.1, was (56.5-11.9% APE), i.e. 44.6% APE.

One of the reasons in initially trying fuzzy logic was its success in building control systems. One control system application is that of tracking a “signal” in order to keep a system under control (Jang et al 1997, Tanscheit and Scharf 1988). This situation is analogous to tracking costs as product or process features change value, rather than time.

The concepts of expert driven and data driven mean that fuzzy logic can be applied with or without data, i.e. at any stage of the product life cycle.

### **6.5.2 Validation: Existing Applications of Fuzzy Logic Related to Costing Have all Been Relatively Successful.**

A process of validation is introduced to the experimental results and also addition to the experimental results, is made through a case study search of the literature as shown in Appendix A2. For example, the ability of a fuzzy logic clustering method to model costs was shown by Pedrycz et al (1999) to be successful when data was scattered. A non-linear



model has data scattered in comparison to linear models, and it has been confirmed in this research that fuzzy logic scattering, in this case subtractive clustering, is a successful modelling tool. The success shown by the low APE in the best models in Experimental Trials 8, for example, and a consideration of fuzzy logic applications through numerical experiments and literature based examples can be summarised.

1. Fuzzy logic was tested on a linear and non-linear model with differing effectiveness.
2. Fuzzy logic is used mainly in control systems where an accurate response is required (Yen et al 1995).
3. There are many other applications of fuzzy logic including decision making, ranking alternatives, expert systems and arithmetic where vagueness, ambiguity and imprecision are modelled.
4. Fuzzy logic can be used for both expert based judgement and input output data pairs or a hybrid of both.
5. Clustering methods are more appropriate for scattered data and can be used to induce meanings or relationships from it.
6. The applications in Appendix A2 are for example: chemical plant capital investment costing, inventory ordering costing, software engineering effort costing, cost modelling of waste in incineration plants, and discounted cash flow problems.

A set of literature based cost models provided information on the problems faced by other researchers:

- The formation of fuzzy sets, and hence capturing the correct expert opinion, is a fundamental problem for using fuzzy logic. Mitaim and Kosko (2001) explored



meaning by using  $\text{sinc}(x)$  functions as membership functions. These were very strange shapes with many lobes, making it difficult to attribute meaning by intuition.

- Turunen et al (1984) applied fuzzy logic to cost estimating the capital costs of chemical plants, in particular they used it to estimate a cost related factor called the Lang factor. The main problem in cost estimating chemical plants was the inexact science of the use of subjectivity. The main issues in fuzzy logic were identified as the ability to include and computerise the subjectivity, for example through linguistic statements of rules.
- Wiehn et al (1996) applied fuzzy logic to cost modelling incineration plants. Uncertainty was a major problem, for example inaccuracies or numerical uncertainty, in quantities related to plant design, and in costs related to a lack of information or quality of specifications.
- Vujosevic et al (1996) approach the Economic Order Quantity problem in different ways of handling imprecision, all producing different results. In particular (1) fuzzification of existing formula, followed by fuzzy arithmetic, then defuzzification, (2) fuzzy arithmetic of discretized points followed by defuzzification, and (3) a fuzzy number comparison method. The problem is that there is no clear choice of which way to choose, but reference to a rule of thumb is made that indicates uncertainty should be removed from the model (e.g. by defuzzification) at the latest stage possible.
- Dohnal et al (1996) use fuzzy logic to capture and integrate the separate data sources of general heuristics, the more accurate cost records and literature based knowledge into one model, for the early cost estimation of capital investment in chemical plants. Hence the problem of data fusion is solved.



- Large rule bases introduce problems with data collection. In addition Kosko (1992) found that very large rule bases tended to produce a very imprecise aggregated fuzzy set that led to particular problems with defuzzification, i.e. different fuzzy logic systems with lots of rules could produce comparable results.

### **6.5.3 Cost model Development Process, Data Identification, Data Collection and Data Analysis: Advantages and Disadvantages of Using Fuzzy Logic**

Fuzzy logic is both a data driven and an expert knowledge driven approach. It has been shown that expert driven approaches are satisfactory for modelling systems in applications such as control (Ross 1995, Jang et al 1997). Therefore the cost model development process can be radically affected by the use of fuzzy logic in utilising expert judgement in a structured, robust and effective manner (the cost model development process is defined in Section 2.2).

Since expert driven cost model development relies on data collection methods with an appropriate qualitative data driven stance, so it is observed that fuzzy logic operates on a restricted set of data collection methods. These methods cost less to operate than the methods typically used to collect quantitative data because of the reduced effort in measurement and storage. For example an interview with an expert will cost the time needed to elicit his opinion, and also lost opportunity costs because of time lost at work. In comparison quantitative data:

- (1) needs a number of data points to become statistically meaningful,
- (2) can have any number of dimensions limited by the number of cost drivers,

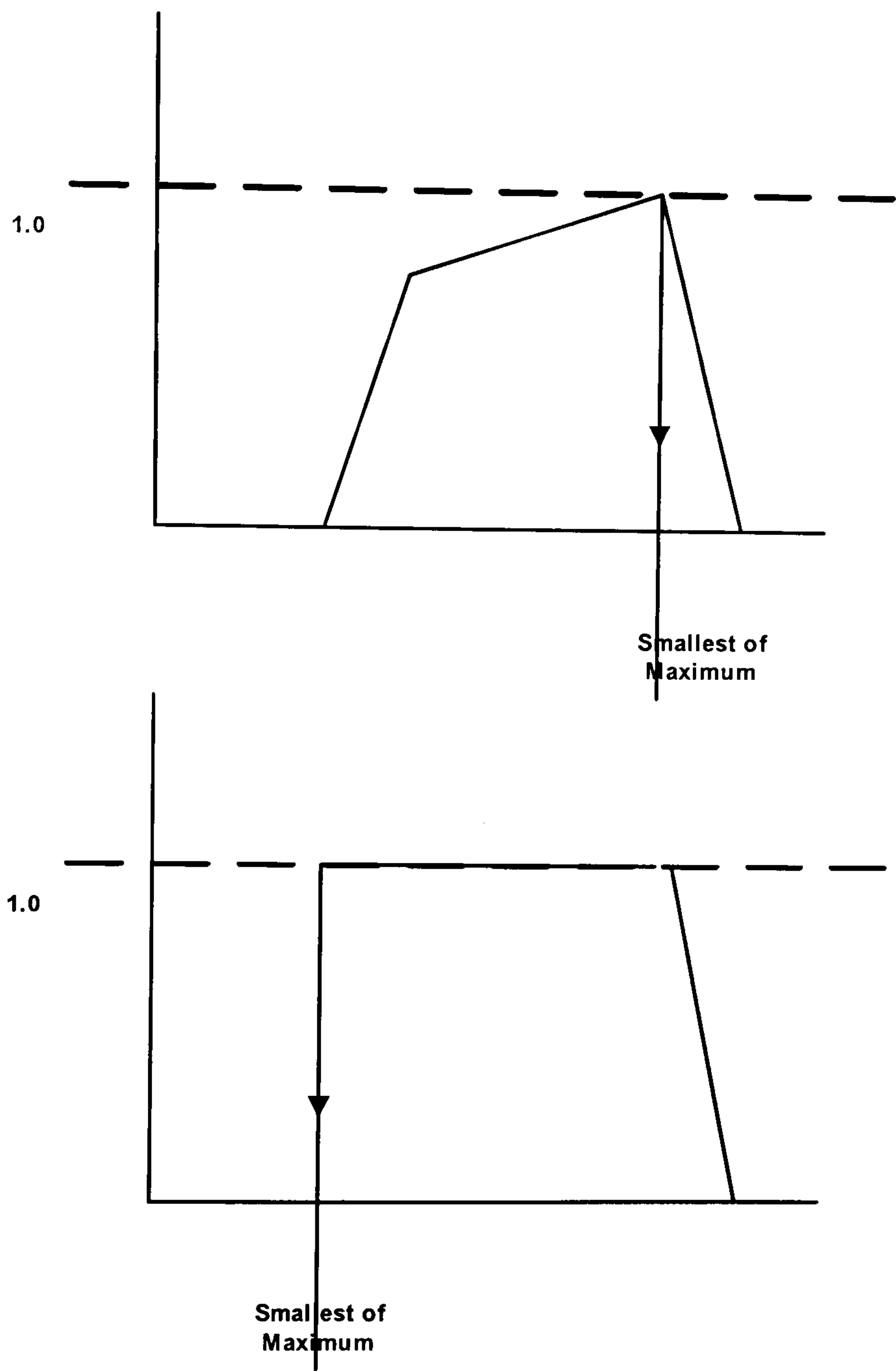


- (3) involves several measurements by hardware or software utilised by personnel,
- (4) occurs over a certain time frame, and
- (5) requires storage, formatting and processing through an Information Technology infrastructure to ensure its relevance.

Expert opinion can be used about the fuzzy logic method itself. It has been shown by the experiments in Experimental Trials 1 and 2, and also Experimental Trials 3 and 4, that the use of different defuzzification methods can produce radically different results. Subjective notions can be held about defuzzification methods that might also influence their choice. For example it is possible to envisage a situation where inexplicable step changes in predicted costs occur in maximum based defuzzification methods. This is shown in Figure 6.4 where a subtle change in the aggregated outputs caused by the fired rules, causes a surprising change in costs. It might be the case that maximum based defuzzification methods be avoided or that their limitations taken into account, despite their improvement in accuracy as shown in Experimental Trials 2 and 4.



**Figure 6.4: Maximum Based Defuzzification Methods are Capable of Unpredictable Results**



Some concerns of using fuzzy logic for cost model development can be summarised through the following observations.



1. Data collection methods can rely solely on the elicitation of qualitative variables and subjective judgement. Qualitative variables like value and quality and relationships between them can be expressed through fuzzy logic (Abbot 1996).
2. The extension principle and fuzzy arithmetic allows fuzzification of crisp functions (Vujosevic et al 1996, Appendix A1).
3. Existing data analysis methods can be fuzzified, e.g. fuzzy regression analysis (Chen and Chang 2002).
4. The choice of four dimensions in the input variable made data visualization practically impossible. This is also noted by Jang et al (1997) when testing ANFIS with higher dimensional models.

A number of concerns in using fuzzy logic for cost model development are summarised in Section A6.8

#### **6.5.4 A Proposed Decision Making Methodology for Choosing Fuzzy Logic**

Previous sections in Chapter 6 have discussed the use of fuzzy logic and its relative merits for cost model development. The final section of Chapter 6 introduces a proposed decision making methodology advisor incorporating a number of examples from the literature and the experiments from Chapters 4 and 5, for the guidance of cost engineers in applying fuzzy logic in practice.

*In Chapter 2 it has been shown how cost model development can be achieved through a cost model development process consisting of 3 significant types of tasks: data identification, data collection and data analysis; and studied from the aspects of cost model*



*characteristics, subjective judgement and assumptions. This is clearly a significant step in forming a structured approach to cost model development. Structure can be further imposed through cost model types, for example cost models can be categorised into three major types (Ostwald and McClaren 2003): bottom up (or grass roots), parametric and similarity-based. There are a number of fuzzy logic methods (Jang et al 1997). The Mamdani method, the Takagi Sugeno Kang (TSK) method, subtractive clustering, and ANFIS were identified in Chapter 3 and used within experiments in Chapters 4 and 5. Further methods include fuzzy arithmetic, fuzzification using Zadeh's extension principle and Fuzzy Cognitive Mapping as identified in Appendix A2. The experiments conducted and reported in Chapters 4 and 5 can be considered alongside a number of these cases from the literature, hence forming two case bases.*

### **6.5.5 Experiment Based**

Section A6.9 records the results of the numerical experiments in a form suitable for reference by the proposed decision making methodology.

### **6.5.6 Rule Base Formation**

A key practical issue for building fuzzy logic models is constructing the rule base in such a fashion as to minimise data collection tasks in the cost model development process. A choice of rule based reduction methods is shown in Section 3.3.2. It is apparent that a potentially large saving in resources can be made by reducing data collection effort, much in the same vein as that described by Maskell and Baggaley (2004) within lean engineering. The emerging application of Type-2 fuzzy logic (Mendel 2001, Section 3.7.3) provides an



essential new method for combining many experts' opinion and hence lever variation in degrees of membership to reduce the rule base and the corresponding number of decisions to build it.

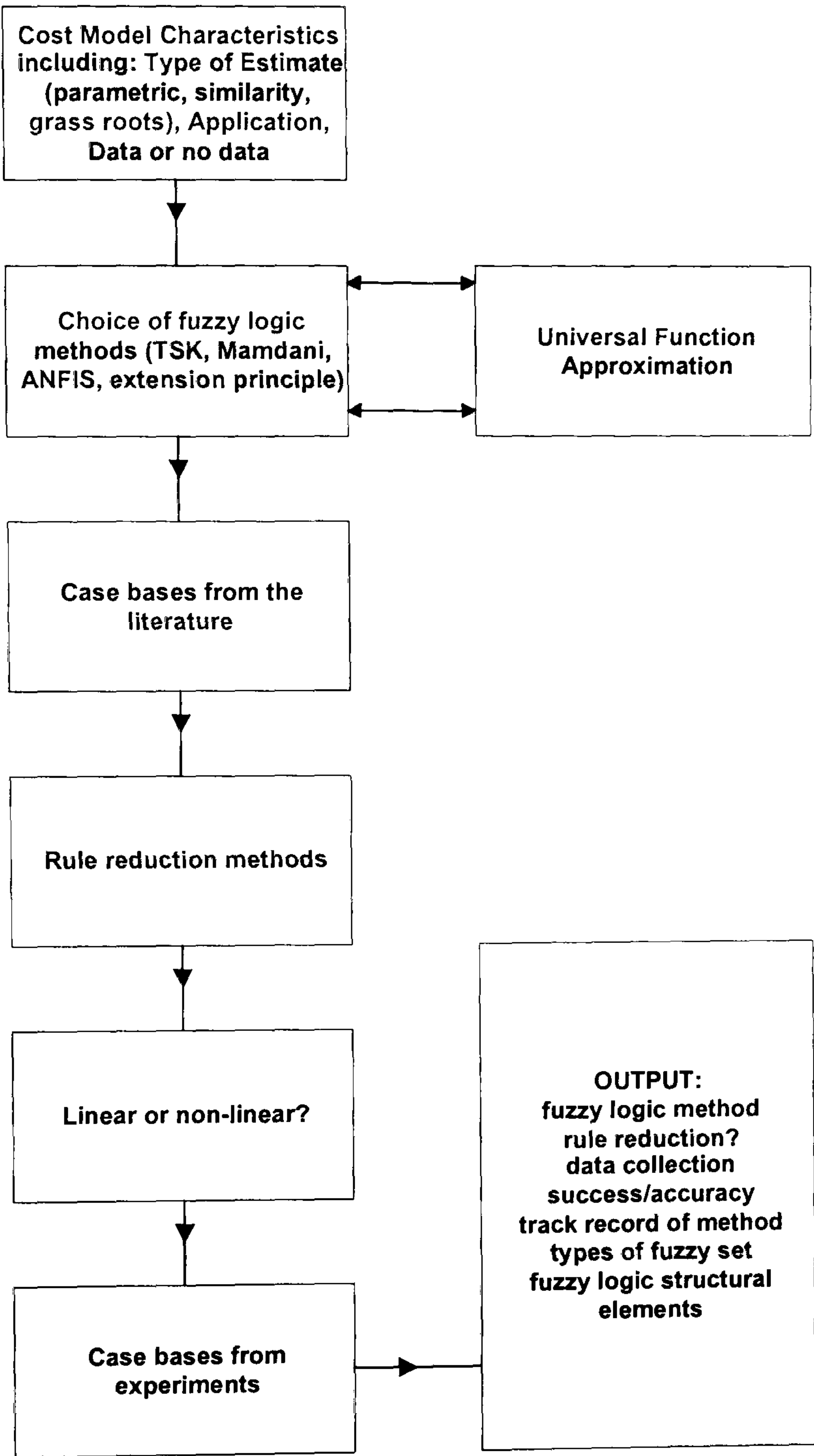
#### **6.5.7 The Proposed Decision Making Methodology for Applying Fuzzy Logic by Cost Engineers**

Work has led to a prototype or proposed decision making methodology. The overall methodology for applying fuzzy logic derived from this research is shown in Figure 6.5. The sources of the components of the methodology are clearly highlighted.

Two case bases are used, namely a literature based case base and an experimental based case base. The literature based case base is used to add to and validate the experimental one in order to provide an overall advice, rather than a proof of performance. The reasons for the different components of Figure 6.11 are shown in Table A6.3. The categories of type of estimate, application, data or no data, and output including the error measure may well be categorised by the cost model characteristics, but they are taken out for particular attention because of their particular importance.



**Figure 6.5: The Proposed Decision Making Methodology for Cost Engineering With Fuzzy Logic**





## **6.6 Advantages and Disadvantages of Using Fuzzy Logic for Cost Engineering:**

### **Summary**

#### **6.6.1 Advantages**

1. No need for knowledge of cost model equation forms.
2. Can work on incomplete information.
3. Models understood by many users.
4. Variables representing “intangibles” can be used, for example “quality” or “fatigue”.
5. It can operate on a diverse number of data sources (expert opinion, cost estimates as cost records, literature (Dohnal et al 1996), diagrams (Leung et al 2003)).

#### **6.6.2 Disadvantages**

1. The “curse of dimensionality” when the number of rules explodes with the number of input variables.
2. Large rule bases are impractical to form, and can detract experts from their formation.
3. Choosing the wrong structure may make for large errors.
4. Automatic model identification methods can produce fuzzy sets with dubitable meaning, and hence may not be useful in this respect.

## **6.7 Sequence of Steps in Using the Proposed Decision Making Methodology Advisor**

1. Choose cost model type dependant on the stage of the product life cycle.
2. Using Chapter 3 (for example Table 3.2) the most appropriate fuzzy logic methods are indicated.



3. Using Table A1.5 the appropriate universal function approximation (also described in Section 3.7.1) proofs are indicated providing information on whether perfect accuracy is possible.
4. Using Appendix A2 then the sample case base of examples from the literature provides important indicators matching the cost model characteristics and previous successes with the method (for example, by similar applications).
5. Section 3.3.2 provides possibilities for rule reduction to reduce data collection costs.
6. Linearity or non-linearity may not be answerable if the form of the cost model equation is not known. If the form is not known precisely the words complex (non-linear) and non-complex (linear) may be used. If the form is known this question provides information for the case base of the numerical experiments in Chapters 4 and 5.
7. Chapters 4 and 5 provide indicators of how accuracy behaves in a non-linear equation, 4.1 and a linear equation 4.2.
8. The output of the methodology advisor is the Level 0, 1 and 2 methods (see Section 6.3 for fuzzy logic structure) most appropriate based on the judgement of the cost engineer as modified by the advisor.
9. The results of the proposed methodology are considered only a starting point in applying the fuzzy logic method to a cost engineering problem.



## CHAPTER SEVEN

### CONCLUSIONS

Changes in the manufacturing environment have placed a new emphasis on cost model development as a predictor of costs. The advent of global competition, mass customisation, short product development times, new technologies, and new materials has meant cost model development processes must produce more cost models, in less time, with less data and be increasingly understandable by a larger variety of users. Fuzzy logic has been identified as a potential method to fill this gap in provision.

The first aim of the research was to:

investigate fuzzy logic methods potentially suitable for use within the cost model development process. In this respect the research has found:

- A rich variety of fuzzy logic methods and structures were available for cost model development. The Mamdani method, the Takagi Sugeno Kang method, the Adaptive-Neuro-Fuzzy-Inference System, and the subtractive clustering method have been specifically tested. These four methods could possibly use a range of data sources, for example imprecise values, expert opinion and historical data points. These are available at different stages of the product life cycle.
- The use of automatic systems identification methods, ANFIS and subtractive clustering, provided a possibility for deriving meaning from large data sets, through interpreting fuzzy sets and rules. These meanings were identified as ambiguous and lacking clarity.

The limitations of this aim found are:

- There are a large number of potential fuzzy logic methods, structures and applications. Only a small subset of these possibilities has been considered by the



numerical experiments. In addition the subset of chosen methods was tested on a limited number of cost model forms. These were described as complex and non-complex. These cost model forms did not comprehensively test the fuzzy logic method as encountered in all requirements for cost model development in industry.

The second aim was to assess the performance of fuzzy logic methods and their possible structural elements. In this respect the research has found:

- Fuzzy logic models could be built for a wide range of accuracies, including models exhibiting values of 50, 12, 5, and 2, Average Percentage Error. It was also found that error could be measured in several different ways which led to a corresponding number of interpretations of the efficacy of the fuzzy logic models.
- A further error measure, the Average Absolute Error indicated the robustness of fuzzy logic by showing that a model with APE of 99.7 has an AAE of 15098, or 4.12% of the output variable range. The clustering method that gave improved results by APE, gave 1082 for AAE, or 0.30% of the output variable range.
- Fuzzy logic exhibited robustness in modelling imprecision in predictor variables and costs, so that an imprecision of 150% of the variable range could produce an Average Absolute Error of 15,376, or 4.19% of the output variable range.
- The automated systems identification methods of ANFIS and the subtractive clustering based method improved their accuracy with a corresponding increase in data points for the modelling process.
- An instance of a “best model” used for estimating a non-linear relationship improved its estimating accuracy once tested on a range of linear cost models chosen using the Taguchi methodology.



- Multiple linear regression analysis was a poor estimating tool for non-linear relationships when compared with the best results of the fuzzy logic methods tested, i.e. Mamdani, ANFIS and the subtractive clustering based method. Multiple linear regression analysis was a clearly better estimating tool than the instance of the Mamdani method used for a specific linear model.
- The use of fuzzy logic improves the usability of cost models by all levels of the business. This is principally because of the use of linguistic variables and rules.

The limitations of this aim are:

- The structures were tested on cost model forms that were both representative of typical cost models, but also can be considered as two finite examples in an infinite number of other potential cost model forms. This infinity of other cost model forms were not explicitly tested.
- The numerical results were from simulation of expert opinion and costs based on arbitrary deterministic equations. They were therefore not subjected to the noise in data, information and knowledge from manufacturing environments. In particular, the effect of human experts themselves on the accuracy of cost models built using their experience, was not quantified.

The first two aims provide the basis for a third aim:

to provide a systematic approach to developing fuzzy logic models in the cost model development process. In this respect the research has found:

- This research has examined cost model development through the structure of data identification, data collection and data analysis methods in conjunction with cost model characteristics and the increased leverage of subjective judgement as an emerging important new data source for cost model development. In particular



fuzzy logic has been identified as a method which can model costs using expert opinion, but also historical data.

- Fuzzy logic has been structured into 3 levels including fuzzy logic methods and fuzzy logic structural elements. The structure is required to facilitate a systematic method of choice in building a fuzzy logic cost model. The systematic method of building fuzzy logic cost models has been made through (1) the use of the Taguchi methodology and also (2) a proposed decision making advisor.
- The results of the experiments in the research produced a ranking of methods, i.e. ANFIS, the subtractive clustering based method, and Mamdani, with ANFIS being the first or most accurate for the APE and AAE error measures. It is not apparent whether this is an absolute ranking, or subject to change with further experiments.
- An efficient and systematic method of designing fuzzy logic models, namely the Taguchi methodology, was used. It was found using the Taguchi methodology that, on occasion, interactions between fuzzy logic structural elements occurred that contributed an additional effect towards error. Hence interactions are an important aspect of the use of the Taguchi methodology in designing fuzzy logic models. It is also inferred that interactions between fuzzy logic structural elements play a part in the fuzzy logic theory.
- A decision support methodology for advising how to build fuzzy logic cost models was proposed including the results of the Taguchi methodology and a sample case base of examples found in the literature. The decision support methodology acknowledged that the process was the possible basis of an indicator only and not an absolute proof of accuracy through fuzzy logic structures.



- The cost model development process using fuzzy logic was structured using the cost model characteristics, data identification, data collection and data analysis tasks. Additional information used within the decision making methodology was also cost model type, presence of a literature based universal approximation proof, error measure type, application and data type for the decision support methodology advisor.
- A large body of research regarding fuzzy logic and cost modelling has been gathered into one place. The principal reason is a proposed fuzzy logic decision making advisor that includes an informal case based reasoning. Its function is to provide validation material and previous experience in advising cost engineers into which fuzzy logic structure to use in which circumstance.

The limitations of this aim are:

- The structured approach is not as secure in the same sense as, say, a mathematical proof, hence cannot be claimed to be beyond error. The process of choice of fuzzy logic is improved, albeit that an element of subjectivity still remains. The technical aspects of fuzzy logic are not entirely hidden, so that the process could be improved by introducing more “cost engineering domain knowledge” terms, for example through the idea and concept of an ontology.



## CHAPTER EIGHT

### FURTHER WORK

The work has highlighted several areas for further research:

- The numerical experiments and decision making methodology advisor can be expanded in scope by considering more methods and structural elements, for example the bounded sum operator, fuzzy arithmetic or parametric defuzzification operators.
- Fuzzy logic is a method identified as reducing data collection. Further data reduction occurs through rule reduction methods. The systematic testing of the efficacy of these methods shall provide information about trade-offs between accuracy and data reduction through rule reduction. For example the hierarchy method can introduce an additive cumulative error for the whole model comprised of the error of several sub-models.
- The numerical experiments can be expanded in scope by considering a range of cost model equation forms by again using the Taguchi methodology.
- This research has considered imprecise quantities through the use of symmetrical fuzzy sets in the numerical experiments. Further work should identify other ways in which experts interact with the fuzzy logic method and their impact on accuracy. The methods include knowledge acquisition methods for use with expert judgement.
- It has been suggested that the accuracy in using fuzzy logic structural elements can be destroyed by the presence of interactions between these elements. In a similar fashion the subjective notions about individual fuzzy logic structural elements could be erroneous when interactions occur. Further work should introduce better



understanding of these interactions within a systematic framework to therefore better understand the process of subjective choice of fuzzy logic structural elements.

- Type-2 fuzzy logic is an emerging application of an originally older idea in which degrees of membership to fuzzy sets are fuzzy sets themselves. Further work shall investigate the accuracy of this new application within the decision making methodology advisor framework. A particular advantage of this new method is its treatment of team-based expert opinion.
- Further work is required to investigate how changing the number of predictor variables within a fuzzy logic cost model affects the accuracy.
- Further work is needed to systematically quantify the robustness of fuzzy logic methods and structures under changing cost model forms.
- Further work should improve the treatment of imprecision by investigating the effect of lessening imprecision by covering the range of predictor variables and costs with several fuzzy logic models rather than altering the imprecision of one model's fuzzy sets.
- Visualisation of cost models' responses is not possible with models with a higher dimension than three. Within the reduced number of dimensions further work shall investigate the cost model error using a subjective evaluation of plotted cost model response as an error measure. In this way the analogy of cost modelling with fuzzy logic, with the modelling of physical systems with fuzzy logic, is improved.
- ANFIS and subtractive clustering automatically convert data sets into fuzzy logic structural elements. Further work should more fully attempt to understand the meaning, if any, of these automatically derived models.



- In the e-commerce era, there is a need for computerisation of the fuzzy logic method in a cost engineer friendly fashion. Subsequently the computerised method must be integrated or made interoperable with other systems.



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# APPENDIX ONE

## FUZZY LOGIC MATHEMATICS

### A1.1 Formal Definition of a Fuzzy Set

$\mu_A : \mathfrak{R} \rightarrow [0,1]$  is the membership function,  $\mu$  that maps members of set A (real numbers) on the Universe of Discourse, to the closed interval in the real numbers between 0 and 1.

### A1.2 Structure of the Four Methods of Mamdani, Takagi Sugeno Kang, Adaptive-Neuro-Fuzzy Inference System, and the Subtractive Clustering Based Method

The structure is provided of the Mamdani, Takagi Sugeno Kang (TSK), Adaptive-Neuro-Fuzzy-Inference System (ANFIS) and subtractive clustering based methods via the fuzzy logic structural elements used for the AND and OR connectives and for the operations of implication, aggregation, and defuzzification.

**Table A1.1: Mamdani Method Structure**

AND	Min
OR	Max
Implication	Min
Aggregation	Max
Defuzzification	Centroid



**Table A1.2: Takagi Sugeno Kang Method Structure**

AND	Product
OR	Probabilistic Operator
Implication	Min
Aggregation	Max
Defuzzification	Weighted Average

**Table A1.3: Adaptive Neuro Fuzzy Inference System Method Structure**

AND	Product
OR	Probabilistic Operator
Implication	Min
Aggregation	Max
Defuzzification	Weighted Average

**Table A1.4: Subtractive Clustering Based Method Structure**

AND	Product
OR	Probabilistic Operator
Implication	Min
Aggregation	Max
Defuzzification	Weighted Average

**A1.3 Formal Definition of Defuzzification Methods**

In all examples  $\mu_A(z)$  is the membership function of set A over the Universe of Discourse of z

Centroid of Area

$$COA = \frac{\int_z \mu_A(z)zdz}{\int_z \mu_A(z)dz} ,$$



### Bisector of Area

$$\int_{\alpha}^{zBOA} \mu_A(z) dz = \int_{zBOA}^{\beta} \mu_A(z) dz, \text{ i.e. the bisector of the overall output membership function shape}$$

(that ranges between alpha and beta) in terms of: there is an equal area to the left and right of the bisector.

### Mean of Maximum (MOM)

$$z^{MOM} = \frac{\int z dz}{\int_{z'} dz}, \text{ where } z \text{ are the maxima}$$

## A1.4 Formal Definition of Some Fuzzy Logic Operators

Let  $\mu_A(x)$  and  $\mu_B(x)$  be the membership function values of  $x$  in fuzzy sets  $A$  and  $B$ .

### Minimum (Min)

$\mu_A(x) \min \mu_B(x)$  is the smallest value of both  $\mu_A(x)$  and  $\mu_B(x)$ ,  $\forall x$ . Min or minimum is the fuzzy logic equivalent of the intersection of 2 sets in classical set theory.

### Maximum (Max)

$\mu_A(x) \max \mu_B(x)$  is the largest value of both  $\mu_A(x)$  and  $\mu_B(x)$ ,  $\forall x$ . Max or maximum is the fuzzy logic equivalent of the union of 2 sets in classical set theory.

### Product (prod)

$\mu_A(x) \text{ prod } \mu_B(x)$  is the multiplication of  $\mu_A(x)$  and  $\mu_B(x)$ ,  $\forall x$ . Prod or product is also known as the scaling operator as it scales down membership functions because of the multiplication together of 2 numbers less than or equal to 1.



## **A1.5 Mathematical Description of Some Popular Membership Functions**

Normal or Gaussian

$$\mu_{A_i}(x) = \exp\left[-\left(\frac{x - c_i}{\sigma_i}\right)^2\right], \text{ where } x \text{ is the variable, } c \text{ is the mean of the normal}$$

distribution, and  $\sigma$  is the standard deviation, for the  $i^{\text{th}}$  fuzzy set,  $A_i$ .

Triangular

$\tilde{a} = (l, m, u)$ , where  $m$  is the apex of the triangle and  $l$  and  $u$  are the end points of the triangle

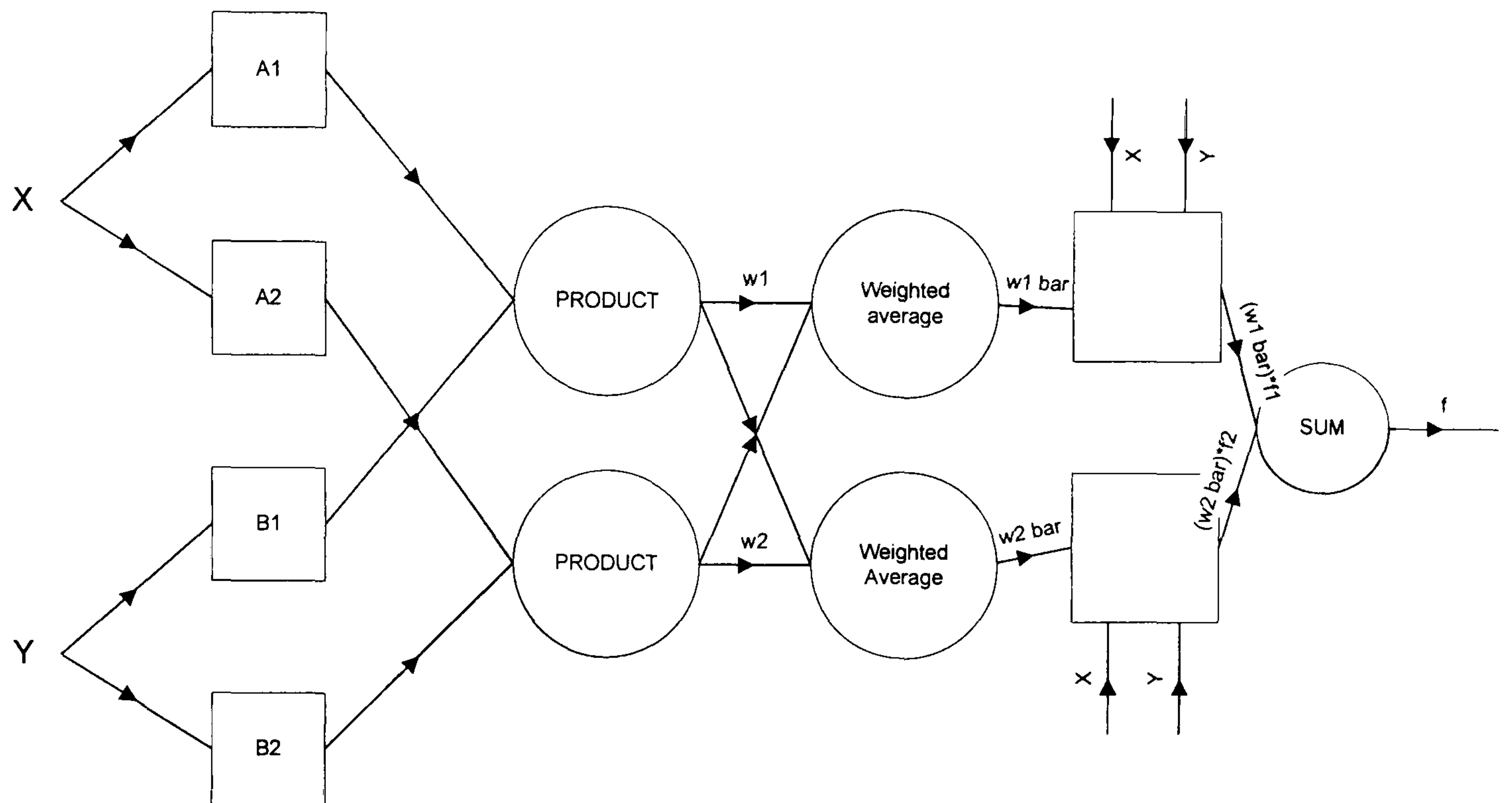
## **A1.6 Alpha Decomposition**

Alpha decomposition involves breaking down a fuzzy set into different levels of uncertainty, called alpha levels. For example  $\alpha=0.5$  identifies the points in the Universe of Discourse for a fuzzy set, for which the membership degrees are 0.5.



## A1.7 Structure of Adaptive Neuro-Fuzzy Inference System

Figure A1.1 ANFIS Structure



Where,

X and Y are input variables' values with membership functions of Variable X being A1 and A2, and the membership functions of Variable Y being B1 and B2.

PRODUCT is the product operator between the linked degrees of membership in the network.

f1 and f2 are output polynomials for the TSK structure ( $f1 = p1x + q1y + r1$ ,

$f2 = p2x + q2y + r2$ , p, q, r1 and r2 are constants), and the output of the model, f:

$$f = \frac{w1f1 + w2f2}{w1 + w2} = \overline{w1}f1 + \overline{w2}f2$$



**A1.8 Sample of Universal Function Approximation Proofs**

**Table A1.5. Universal Function Approximation Proofs**

Universal Function Approximation Proof	Issues
Nguyen et al (1996)	Covers all universal function approximation theory to 1996. Uses the property of denseness of a function with a supremum norm to prove continuity.
Wang and Mendel (1992)	Control function approximation to an arbitrary accuracy on a compact set using membership functions that are at all places positive, the output fuzzy sets are crisp, the product method used for AND, and the centroid defuzzification method. The input membership functions are: $\mu_x = \mu_x(kx + l)$ , for $k \neq 0$ and $k$ and $l > 0$ , $x$ is the Universe of Discourse.
Cao et al (2001)	Mamdani control strategy is a universal controller (i.e. “a fuzzy control law which can stabilize a given complex non-linear system if the system can be stabilized”). The fuzzy logic structural elements used are: “centre-average defuzzifier, product inference, and singleton fuzzifier”)
Nguyen et al (1996)	An arbitrary fuzzy control methodology can approximate arbitrary continuous functionals. Nguyen (1996) states the number of correct rules is finite.
Wang (1992)	Fuzzy logic methodology of product for the "AND", input membership functions are gaussian, output membership functions are crisp, and defuzzification is the centroid. method.
Kosko (1994)	Standard Additive Model (SAM)
Nguyen et al (1996)	Fuzzy methodology is minimum is the "AND", input fuzzy sets are Gaussian, output fuzzy sets are crisp and the centroid is the defuzzification method.



Table A1.5: Continued

Universal Function Approximation Proof	Issues
Nguyen et al (1996)	A “realistic” fuzzy logic methodology of an arbitrary choice of “AND”, “OR”, defuzzification method and membership functions. “Realistic” means all membership functions are continuous, are positive in some range and zero in the rest of the range.
Jang et al (1996), Jang (1993)	ANFIS using the Stone-Weierstrass theorem.

“fuzzy controls can approximate an arbitrary real continuous function,  $f(x)$  with arbitrary accuracy” (Nguyen et al 1996).

**A1.9 The Formal Definition of Zadeh’s Extension Principle (Jang et al 1996)**

$$A = \mu_A(x_1)/x_1 + \mu_A(x_2)/x_2 + \dots + \mu_A(x_n)/x_n$$

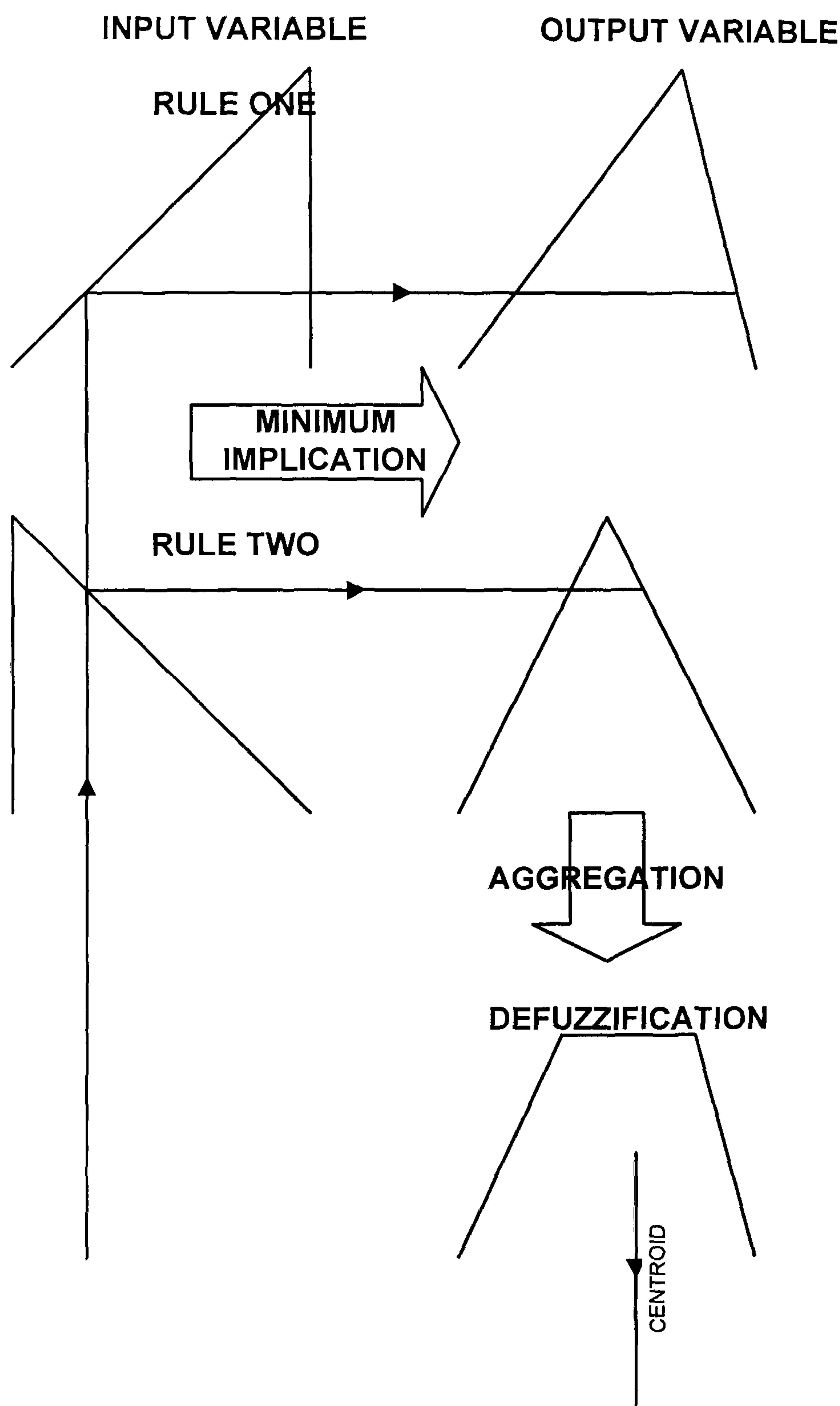
$$B = f(A) = \mu_A(x_1)/y_1 + \mu_A(x_2)/y_2 + \dots + \mu_A(x_n)/y_n$$

$$\mu_B(y) = \max_{x=f^{-1}(y)} \mu_A(x), \text{ where A and B are fuzzy sets.}$$



**A1.10 Example Interactions within a Fuzzy Inference System**

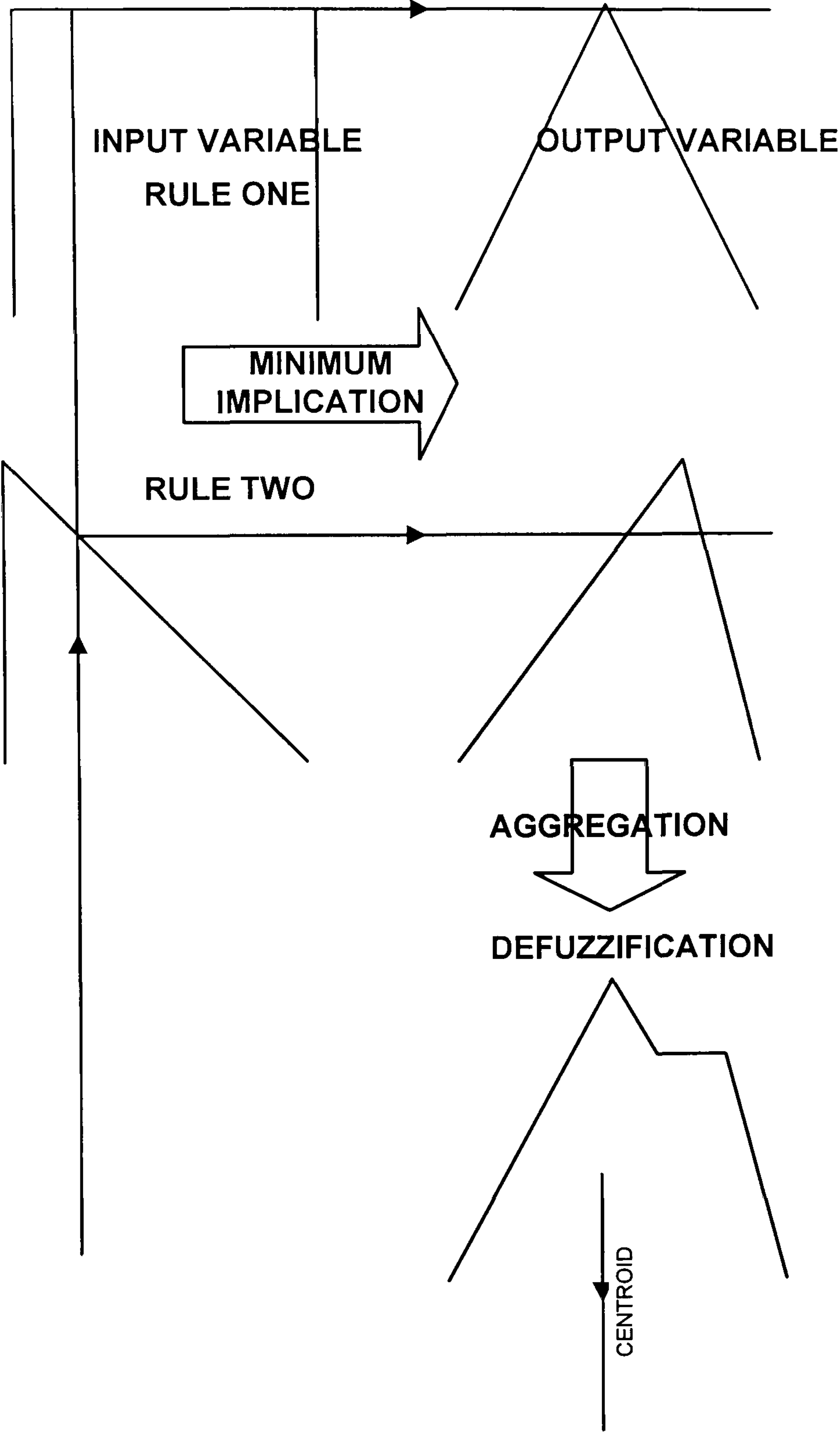
**Figure A1.2: Interaction Between Fuzzy Set Shape and Defuzzification, Example One**



Figures A1.2 and A1.3 show how changing fuzzy set shape in the input variables has a clear interaction with the centroid defuzzification method.



**Figure A1.3: Interaction Between Fuzzy Set Shape and Defuzzification, Example Two**





**APPENDIX A2: A CASE BASE OF EXAMPLES FROM THE LITERATURE TO BE USED WITHIN THE DECISION MAKING METHODOLOGY**

	Dohnal et al (1996)	Wiehn et al (1996)	Wen and Lee (1999)	Irani et al (2002)
Application	Chemical Plant Capital Cost	Cost modelling of waste in incineration plants	Cost function for wastewater treatment systems	Evaluation of Information Technology/Information Systems from a financial perspective
Product Life Cycle Stage	Concept	Design stages	Early design	Concept evaluation
Subective Judgement	"complex, vague, integrated, relatively ill known" "engineering feeling, gossip" "human ability to draw conclusions which are based on analogy"	"uncertainty inherently associated with the variation of the environment", linguistic variables	Uncertainty, vague phenomenon, effects of human estimation, errors considered through fuzziness rather than randomness	-



	Dohnal et al (1996)	Wiehn et al (1996)	Wen and Lee (1999)	Irani et al (2002)
Variables	Capital cost of a plant, Cost of main equipment, Lang factor based on earlier projects, Scope of estimate, Process category, Average cost of equipment items, Process complexity	throughput, excess air, flow velocities, scrubber efficiencies, number of plant units	Plant construction cost, operation and management cost, design flow rate, treatment degree, collection area, influent	Categorised under strategic, tactical, operational, and financial levels
Novelties	Consistency check of rule bases (how good is the knowledge?). How similar are the conditional statements?	Sensitivity analysis allows variation of several variables at once	-	An ability for the cost model to function with changing knowledge
Method	-	Fuzzy numbers	Fuzzy linear regression	Fuzzy Cognitive Map (FCM)
Sub Method Level 1	-	Generation of fuzzy numbers using Monte Carlo simulation	Fuzzy numbers, Fuzzy regression analysis and Zadeh's extension principle	Learning methods



	Dohnal et al (1996)	Wiehn et al (1996)	Wen and Lee (1999)	Irani et al (2002)
Sub Method Level 2		Use of alpha cuts	Linear programming	-
Fuzzy logic structural elements	Fuzzy sets, minimum intersection, maximum aggregation, centre of gravity defuzzification	-		-
Fuzzy sets	Trapezoids using parameters a,b,c,and d	Piecewise continuous from the alpha-cuts	Symmetrical Triangular fuzzy sets used as parameters within the fuzzy regression	-
Rules	General and specific. There are 113 rules (81 general heuristics, and 31 specific heuristics)	-	N/A	-
Fuzzy operators	Maximum and minimum (the so called max min)	-	Product operator for cartesian product of the fuzzy regression parameters	-



	Dohnal et al (1996)	Wiehn et al (1996)	Wen and Lee (1999)	Irani et al (2002)
Defuzzification Method	Centre of gravity (silhouette), Centre of gravity (multiple)	-	N/A	Causal rules between, say, numbers, objectives and policies, as part of a structure that is based on the human brain. Difficult to quantify relationships between strategic, tactical, operational and financial considerations. Expert rules.
Accuracy	The method is not explicitly measured by an error metric.	-	The method is not explicitly measured by an error metric.	The method was only a proposed methodology
Success	The method is not explicitly measured by an error metric.	"the cost calculation based upon fuzzy sets proves to be a favourable tool in assisting decision making under uncertainty"	The model explained 50% variation from actual values by the concepts of fuzziness through the fuzzy linear regression method	The method was only a proposed methodology



	Dohnal et al (1996)	Wiehn et al (1996)	Wen and Lee (1999)	Irani et al (2002)
Manufacturing Volumes	One-off capital cost of a plant	One-off capital cost of a plant, number of historical plants in the model was 67	One-off capital cost of treatment plant	-
Data identification methods	Literature sources and cost records, i.e. comparative	Expert rules in the form of modules of a cost estimating software system. Expert input	Previous cost examples from a database	Experts
Data collection methods	Interview and formalised rule construction from cost examples	Literature based data sources and existing plants, collected into a database for storage	Design data collected from 26 municipal wastewater treatment plants placed into a database	Expert interview and knowledge acquisition
Variety of tasks	There are a variety of chemical plants in the research (between 10 and 100)	High variety of tasks constrained by legislation	Quite a variety of input specification parameters	One-off large Information Technology investments



	Dohnal et al (1996)	Wiehn et al (1996)	Wen and Lee (1999)	Irani et al (2002)
Repetitiveness of tasks	Chemical plants are similar up to the point of the same specification variables	A forecast of residual waste invokes plant design parameters that subsequently invoke Cost Estimating Relationships	None	Project based implementation
Amount of subjective judgement	A quote from the research provides some information: “a preliminary investment cost estimation is a very important and vague problem”	“uncertainty that is inherently associated with the variation of the environment”, numerical uncertainty in plant design, quality and amount of available information, seasonal fluctuations for plant capacity	Measured through the concept of imprecision through the width of fuzzy sets	Presence of so-called intangibles (, i.e. benefits difficult to quantify using existing numerical techniques (“human and organisational benefits”))
Personnel whom operate the system	Early designers and planners	Early designers	Decision makers, plant designers	Mainly strategic decision makers, but also impacts tactical and operational level
Detail of input data	Precise high level specifications of chemical plants	Approximate or incomplete values	Can be precise or fuzzy numbers	Ambiguous and imprecise quantities are prevalent, for example “improved productivity”
Estimate application time	The time needed to build fuzzy sets and general and specific heuristics. Also the time needed to check the consistency of the rule base	The time needed to produce several alpha cuts of the output variable, capital investment cost	The time needed to run an automatic method through a database	The time needed to interpret a Fuzzy Cognitive Map



	Dohnal et al (1996)	Wiehn et al (1996)	Wen and Lee (1999)	Irani et al (2002)
Operating Costs	Maintenance of the rule base	The maintenance of previous experience and the need for judgement to build the fuzzy logic system	Maintenance of the database. Reduced operating cost due to the use of fuzzy input values	Easily maintained model with easy to collect input variables
System set-up costs	Experts' time and infrastructure with cost records	The set-up of expert knowledge and software system in order to fully lever the fuzzy logic method	Cost of data collection lowered because of the ability to capture fuzziness, LINDO linear programming software package	Expert time and knowledge



APPENDIX A2: A CASE BASE OF EXAMPLES FROM THE LITERATURE TO BE USED WITHIN THE DECISION MAKING METHODOLOGY

	Chen and Chang (2002)	Yao et al (2000)	Liu, S. and Kao, C. (2004)	Chan et al (2002)
Application	Cost of construction of wastewater treatment plants	Inventory problem without backorder	Transportation in logistics and supply chain. Minimisation of travel costs given supply and demand within a network of nodes. In particular the determination of the membership function of total cost using fuzzy sets for unit transport cost, supply and demand.	Quality improvement, “with the fuzzy decision model, companies are capable of making cost effective implementations decisions and evaluations of process improvement”
Product Life Cycle Stage	Design stages	Production	Production	Production through continuous improvement processes



	Chen and Chang (2002)	Yao et al (2000)	Liu, S. and Kao, C. (2004)	Chan et al (2002)
Subjective judgement	Modelling of human estimation and differences between costs because of differing cost structures	Determining the parameters of fuzzy sets for variables	Production of vague quantities for the variables to be represented by fuzzy numbers	Judgement, lack of precision, “the purpose of this study is to examine the theory of fuzzy sets to demonstrate the applicability of fuzzy logic for expressing the inherent imprecision in the way that people think and make decisions about the quality improvement implementation”, blurred boundaries between classes, “process of human judgement”
Variables	Examples include facility size, levels of waste water treatment and category of plant	Order quantity per cycle, total demand, time of planning period, cost of storing one unit per day, cost of placing a order	Unit shipping cost, quantity supplied by a node, quantity demanded by a node	One goal of “on-time delivery” and two constraints of “failure rate” and “failure cost”
Novelties	Consideration of fuzziness as uncertainty rather than identical and independently distributed random uncertainty for each parameter	Dealing with the complexity of using several fuzzy numbers	The use of a fuzzy set for the objective value of cost in the programming method contains more information rather than just a crisp number.	The use of fuzzy sets for decision making in a continuous improvement culture under Total Quality Management



	Chen and Chang (2002)	Yao et al (2000)	Liu, S. and Kao, C. (2004)	Chan et al (2002)
Method	Least squares regression compared to fuzzy linear regression and fuzzy goal regression	Fuzzification of an existing cost model using the extension principle followed by centroid defuzzification	Fuzzification of linear programming using Zadeh's extension principle and through the process of alpha cuts.	Fuzzy sets for goals and constraints are intersected to produce a fuzzy set that is defuzzified to produce the decision.
Sub Method Level 1	Linear and goal programming methods		Linear programming Zadeh's extension principle.	Defuzzification is by maximum degree of membership
Sub Method Level 2	-		Fuzzy numbers.	-
Sub Method Level 3	-		Zadeh's extension principle	-
Sub Method Level 4	-		Alpha cuts to calculate degrees of membership at discrete points only, i.e. the fuzzy set is calculated "numerically"	-



	Chen and Chang (2002)	Yao et al (2000)	Liu, S. and Kao, C. (2004)	Chan et al (2002)
Fuzzy logic structural elements	-	Fuzzy sets, extension principle, centroid method		Fuzzy sets, intersection, defuzzification
Fuzzy sets	-	Normal triangular fuzzy numbers	Any continuous shape	Concave and convex curved sets. Fuzzy sets are created by a simulation tool called “ithink”
Rules	-	N/A	-	There are no rules as such but a comparison of processes and degrees of membership to quality rating. The process with the least degree of quality rating has the benefit of having resources assigned to it.
Fuzzy operators	-	N/A	-	-
Defuzzification Method	-	Centroid	Via alpha decomposition	Identification of the lowest degrees of membership to a “quality rating” fuzzy sets allows the deployment of resources to increase the rating



	Chen and Chang (2002)	Yao et al (2000)	Liu, S. and Kao, C. (2004)	Chan et al (2002)
Accuracy		<p>“we find that, after defuzzification, the total cost is slightly higher than in the crisp model, however, it permits better use of the economic fuzzy quantities arising with changes in orders, deliveries, and sales”</p>		<p>Using fuzzy sets instead of conventional techniques was tested using a simulation in which the percentage of products conforming to quality produced was 76.5% (conventional techniques) and 90% (fuzzy techniques). It is also stated that, “the fuzzy decision model will result in a 45% saving in production time”, throughput is 17% better using fuzzy logic from the simulation. There are other improvements</p>



	Chen and Chang (2002)	Yao et al (2000)	Liu, S. and Kao, C. (2004)	Chan et al (2002)
Success	“the newly derived fuzzy goal regression model owns the most robust algorithm in practice”	No error figures are supplied but it is noticed that when the fuzzy version approaches the crisp case, there is agreement with this crisp case.	A possibility of producing an incorrect solution is raised for the methodology of programming using fuzzy numbers by Julien (1994) and Parra et al (1999)	“The fuzzy decision model to integrated process performance measurement proposed here produced better competitive results on a simulation of determination of improvement implementation than the conventional approach



	Chen and Chang (2002)	Yao et al (2000)	Liu, S. and Kao, C. (2004)	Chan et al (2002)
Data identification	Comparative estimating through a database	An existing cost model that is fuzzified	-	Performance measurement framework driven by “competitive priorities”. Also a question and answer scheme, e.g. “possible sources of data?”
Data collection	78 cost examples to be stored in a database	Production of fuzzy sets	Production of vague quantities from each node in a network of companies	MRPII system
Manufacturing volumes	One-off capital cost estimation	Various	Determined by threshold values within the model	All
Variety of tasks	Variety of dependent variables	N/A	Only transportation task	All
Repetitiveness of tasks	Non-repetitive variables	N/A	N/A	All
Amount of subjective judgement	Special attention is made to fuzziness rather than randomness as a form of uncertainty caused by linguistic descriptions and human estimation. Vague goals and imprecise parameters	Production of fuzzy numbers, uncertainty through market demand	Production of fuzzy numbers	High



	Chen and Chang (2002)	Yao et al (2000)	Liu, S. and Kao, C. (2004)	Chan et al (2002)
Personnel whom operate the system	High level economic planning of national placing and timing of building waste water treatment plants	Procurement manager	Logistics and supply chain managers	TQM teams, strategic decision makers
Detail of input data	Early design specifications	Precise numbers	Precise number of supply, demand and unit shipping costs	Precise numbers
Estimate Application time	The time to collect specification data to input into the model	Low in comparison to other methods that need crisp values to put into the cost model	The time to collect the fuzzy sets for the model	Low
Operating costs	High to maintain the cost examples on which the regression methods are based	Low due to the use of fuzzification of parameters	Software and expert knowledge in the fuzzy logic method	Low
System-setup costs	High in initial collection of cost examples	Depends on the costs of collecting fuzzy sets and the harnessing of fuzzy logic knowledge	The cost of collecting vague data, and applying the method through method knowledge and software	Low, but dependant on the costs of data collection, i.e. expert time and MRPII operation costs



## APPENDIX FOR CHAPTER 3, A3

### A3.1 The Mathematics of Implication

The implication process modifies the membership functions in the output variable within each rule. Implication is written  $A \Rightarrow B$  and is represented as an “if then” rule, i.e. if A then B. A is called the antecedent and B is called the consequent. The expression,  $A \Rightarrow B$ , becomes an operation between the membership function of A and the membership function of B to produce a two dimensional membership function of  $A \circ B$  where  $\circ$  is an operation between A and B. For example if the operation is the product and  $\mu_A(a)$  is the membership function value for  $a \in A$  and  $\mu_B(b)$  is the membership function value for  $b \in B$ , then:

$$A \Rightarrow B \equiv A * B \equiv \mu_{A*B} = \mu_A * \mu_B \text{ for all } a \text{ in } A \text{ and all } b \text{ in } B$$

$$A = \{(a, \mu_A(a)) \mid a \in A, \mu_A(a) \rightarrow [0,1]\}, B = \{(b, \mu_B(b)) \mid b \in B, \mu_B(b) \rightarrow [0,1]\}$$

making for a 2-dimensional fuzzy set. If the operation is the minimum (min) then  $\circ$  is represented by min. Minimum is the intersection of the fuzzy sets (Appendix A1).

Consider an input value  $a_1 \in A$ . If the operation is the product and  $\mu_A(a_1)$  is the membership function value for  $a_1 \in A$ , then the output membership function B is modified via:

$$a_1 \Rightarrow B \equiv a_1 * B \equiv \mu_{a_1*B} = \mu_{a_1} * \mu_B \text{ for } a_1 \text{ in } A \text{ and all } b \text{ in } B$$

making for a 1 dimensional fuzzy set, as is the overwhelming case in inference found in the literature.



The implication process produces a general rule that is combined with specific observations, i.e. an observation is a specific case of an individual input value into a model (this is usually a vector for the case of more than one input). In theory this input value is generalised to be a fuzzy set (the case of an input value being another fuzzy set has been rarely found in the literature). If the observation is a fuzzy set  $A'$ , the inferred fuzzy set  $B'$  using the implication is:

$$B' = A' \circ (A \Rightarrow B)$$

Where  $\circ$  is an operation between the membership functions,  $A'$  and  $(A \Rightarrow B)$ . This can therefore be rewritten:

$$\mu_{B'} = \mu_{A'} \circ \mu_{A*B}$$

If the fuzzy sets are represented graphically using x and y axes, and if  $B'$  is on the y axis and  $A'$  is on the x axis then

$$\mu_{B'}(y) = \bigcup_x \mu_{A'}(x) \circ \mu_{A*B}(x, y)$$

where  $\circ$  is an operation that can be product or min amongst many. Because the implication function is two dimensional, all the  $x$ 's have to be aggregated using union for each  $y$  to eventually give a fuzzy set that is a function of  $y$ . This is the output fuzzy set of the rule.

In practice the input is a single value, for example  $a_1 \in A$ . To find:

$$\mu_B(y) = \bigcup_y \mu_A(a_1) \circ \mu_{A*B}(a_1, y)$$

where  $\mu_B(y)$  is the inferred output fuzzy set for the rule  $(A \Rightarrow B)$  from the value  $a_1$ .



Aggregation completes the inference process. Here all the output fuzzy sets for each rule are aggregated in order to produce an overall output membership function. The aggregation is done using the union of the sets that is also called the maximum or max. Other operators can be defined that will yield a different overall output membership function.



## APPENDIX FOR CHAPTER 4, A4

### A4.1 Method of Building the Mamdani Method Models

There have been several uses of the Mamdani method in the literature, e.g. control systems, manufacturing engineering, and geology (Ross 1995, Yan et al 1994, Yen et al 1995, Sonmez et al 2004). The steps to build a Mamdani fuzzy logic model in this research are summarised in steps (a) to (d).

- (a) Decide how to make a fuzzy set from an individual data value,
- (b) decide where to place the fuzzy sets that can be made from the individual data values,
- (c) decide how to generate a rule base from all possible combinations of the fuzzy sets in input variables, X1, X2, X3 and X4,
- (d) decide how to defuzzify an aggregated output after inference,
- (e) test the model using 350 test data points to form an Average Percentage Error (APE).

In the following steps (a) to (e) are more fully explained in how they were carried out.

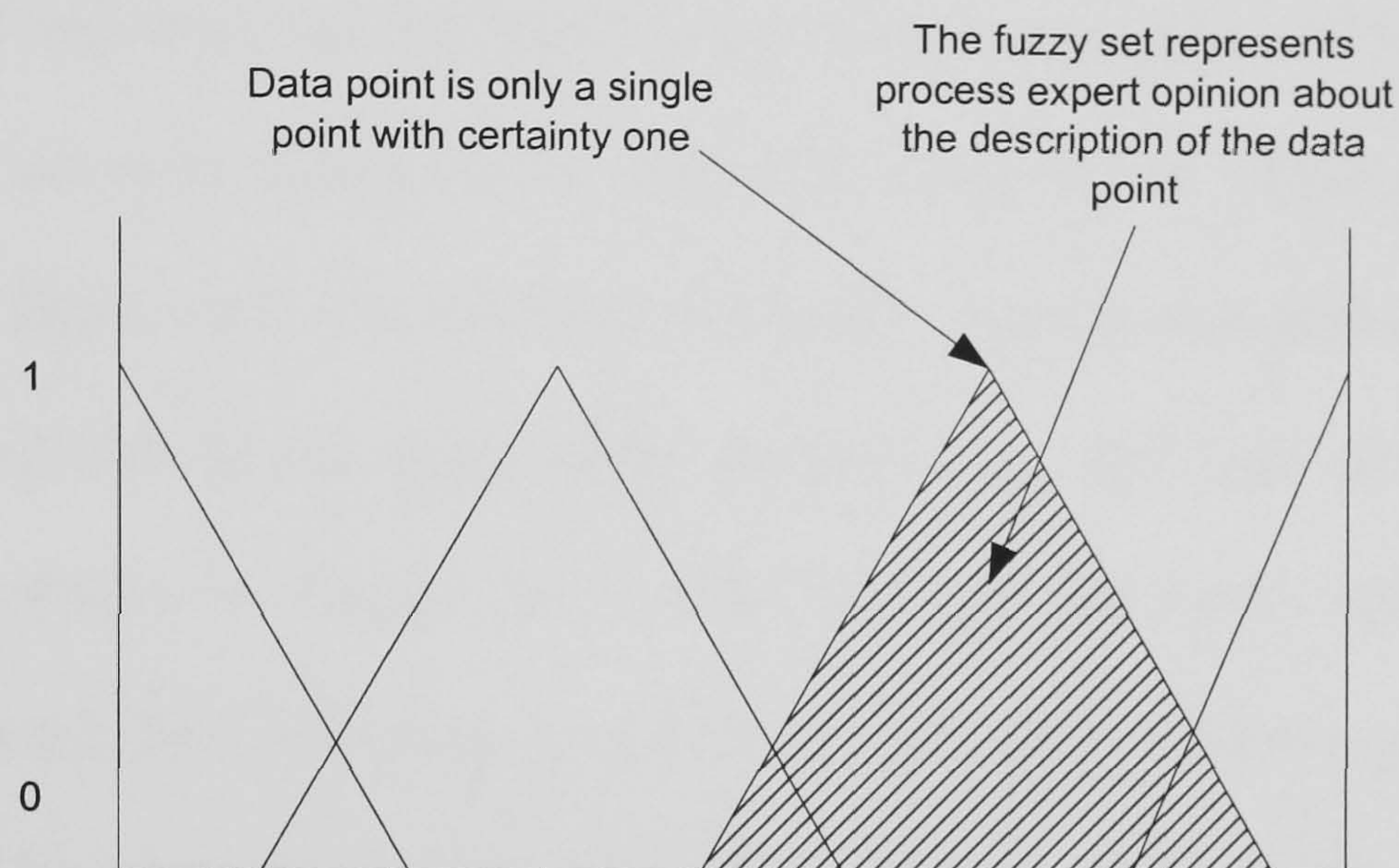
- (a) Hence the first step was to decide how to use the data from equation (4.1) to build fuzzy sets. Precise data generated using Equation (4.1) can be converted into imprecise fuzzy numbers and represented by linguistic variables, i.e. data points are fuzzified. If the data point was decided to be the value, 5, then fuzzification leads to the fuzzy number, “about 5”. Five is made to be “about 5” with certainty 1. All other numbers in the fuzzy set are decided to be “about 5” to a lesser degree, symmetrical around the value, 5 (if the fuzzy set is not symmetrical then this infers some special knowledge has led to “about 5” adopting a different meaning). Figure



A4.1 shows pictorially how a data point from equation (4.1) is used. In effect the equation is used to emulate an ideal process expert, i.e.,

- (i) an expert responds in interview by estimating facts as data points that he cannot describe precisely. An ideal process expert responds with precise data points.

**Figure A4.1: Fuzzification of a Data Point.**



- (ii) The approximate data points have meanings attached to them and consequently become qualitative, e.g. data point  $x = 1$  becomes, “about 1”, that in turn is described by the word, “small” due to attaching approximate meanings. “Small” can be described as a term of the linguistic variable,  $X_1$ . Imprecision is introduced via the shape of the overall fuzzy set, paying particular attention to the location of its range and its symmetry, for instance. It is decided that our ideal process expert responds in interview by specifying a symmetrical fuzzy set about the precise value from equation (4.1). It is noted, however, that (i) an infinite choice of fuzzy sets is possible, and (ii) the many more fuzzy sets that can be chosen are more sophisticated in their range and unsymmetrical in general.



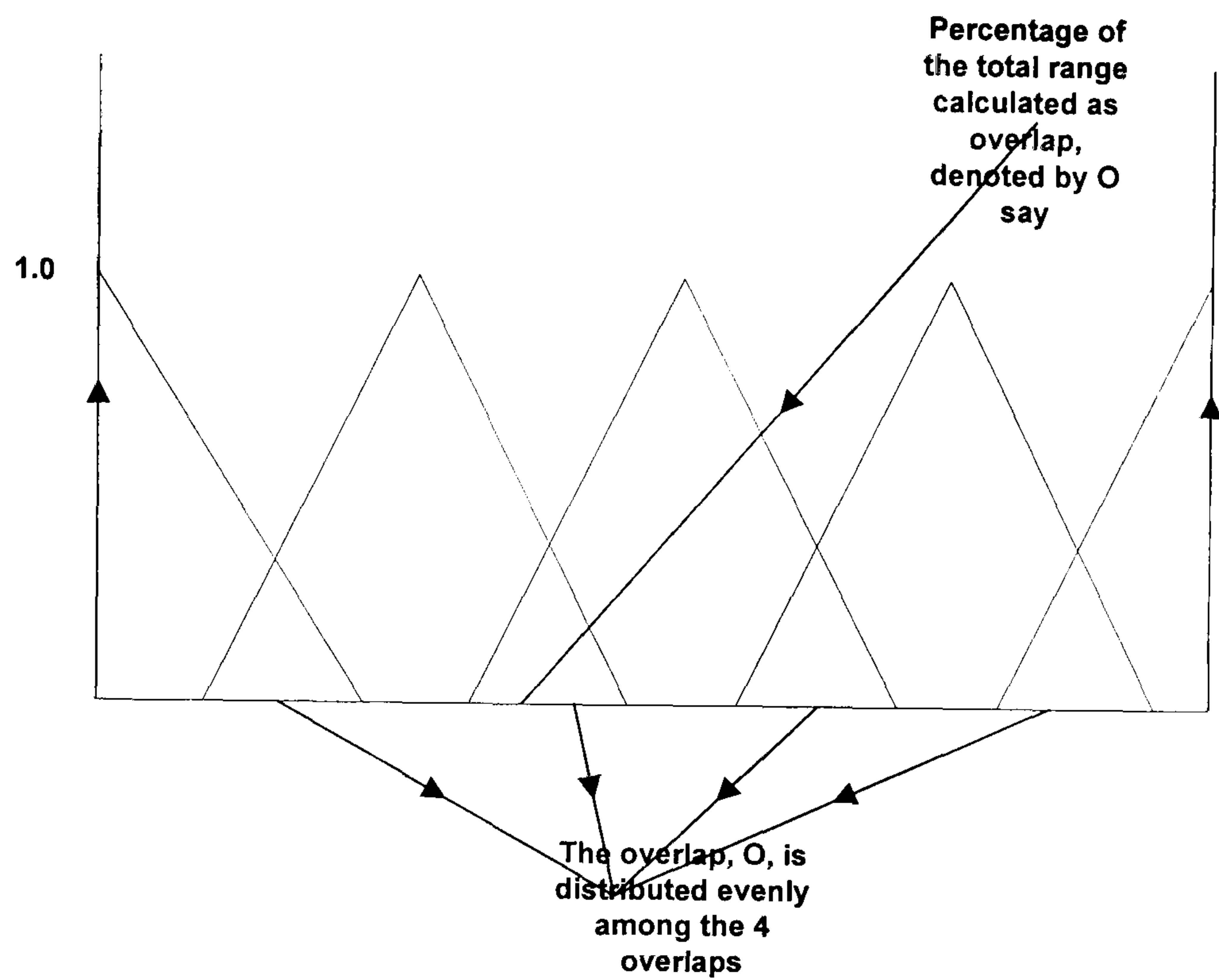
(iii) Data points from input variables become connected via “if then” rules involving the qualitative variables. Equation (4.1) allows this process to be followed artificially, i.e. data points for  $X_1$ ,  $X_2$ ,  $X_3$  and  $X_4$  get connected to  $Y$ . Fuzziness, imprecision, subjectivity or expert opinion is introduced by making the data points into fuzzy sets and the equation connecting inputs to outputs, into rules.

(b) The second step was to decide where the fuzzy sets were going to be. The centres of each fuzzy set were placed evenly along the range of the variable, depending on how many there were. The first and last centre were always placed as coinciding with the extremes of the range of the variable, e.g. only half of the fuzzy set is visible as shown in Figure A4.1. The bases of the fuzzy sets were not so straightforward. The percentage overlap was how much of the range of the variable that was to be covered by overlapping fuzzy sets, i.e. 50% overlap of  $X_1$  meant 2 units of the range between 1 and 5 was covered by overlapping fuzzy sets. The overlap was distributed evenly among the fuzzy sets and was symmetrical, as shown in Figure A4.2.

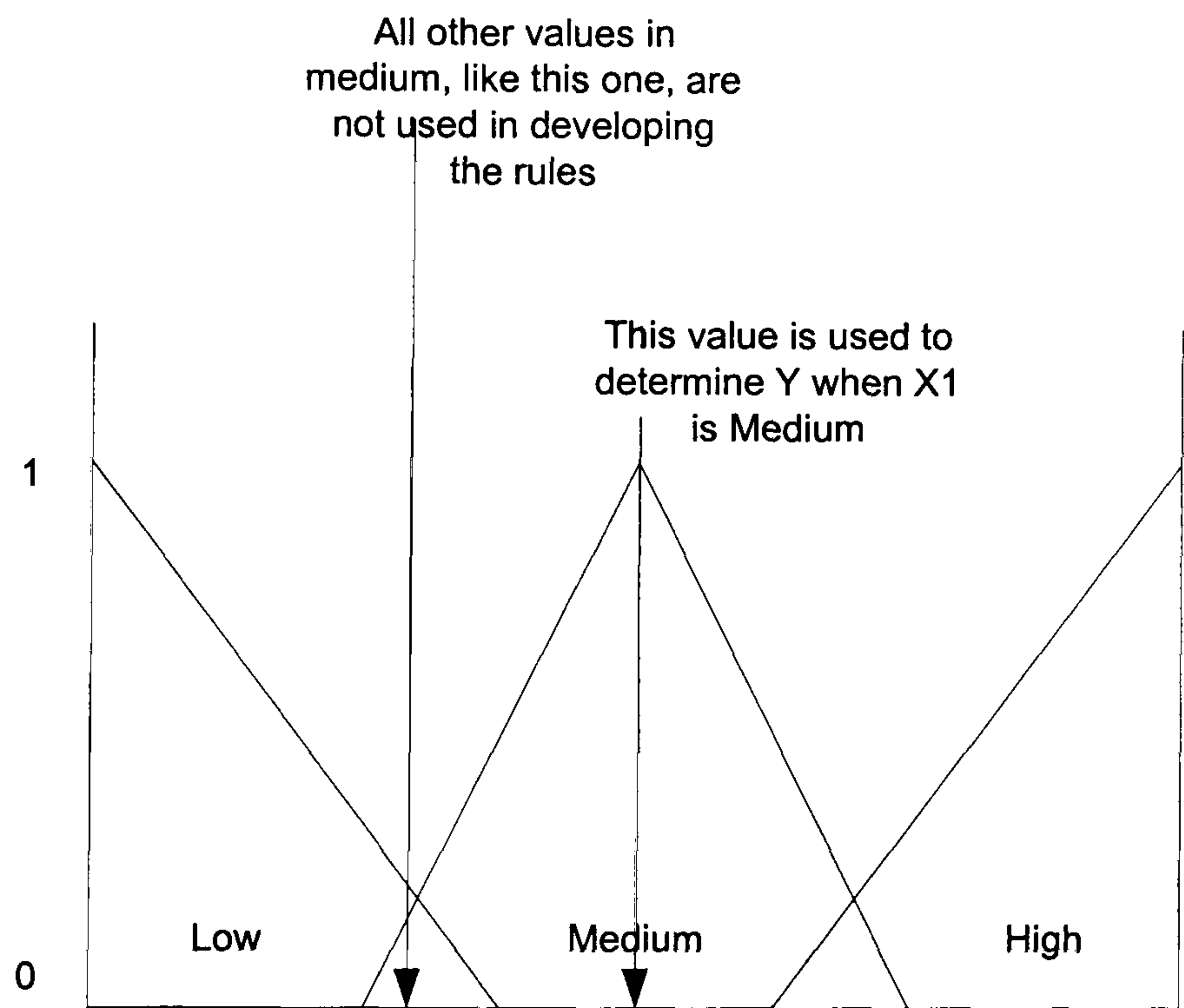


**Figure A4.2: How Overlap is Determined in the Experimental Trials, e.g.**

**Experimental Trials 1.**



**Figure A4.3: Data Points Become Fuzzy Sets.**



(c) The third step was to generate the rules. Equation (4.1) gives accurate data, i.e. a data source from which subjective opinion can be derived. Only actual values with a certainty or truth-value of one in a membership function for that input variable, as



shown in Figures A4.3 and 4.5, are put into the function and a value of Y, as shown in Figure A4.5, is calculated. This value of Y can be found occurring in a membership function for the output variable with a higher degree of truth than any other. Therefore a membership function from the range of Y is chosen that most contains the value calculated from the input values in terms of degree of membership. Therefore the membership function that gave the higher degree of certainty was chosen to complete the rule. The method of choice is shown in figures A4.4 and A4.5. Figure A4.4 demonstrates that only the fixed values of X1, X2, X3 and X4 with a certainty of 1 are considered. A possible generated value of Y from equation (4.1) is shown in figure A4.5. The value is clearly mostly represented by the fuzzy set labelled medium. Therefore the generated rule from figures A4.4 and A4.5 is “if X1 is medium and X2 is high and X3 is low and X4 is medium then Y is medium”.



**Figure A4.4: Values That Have a Certainty of 1 are Used to Generate Values of Y.**

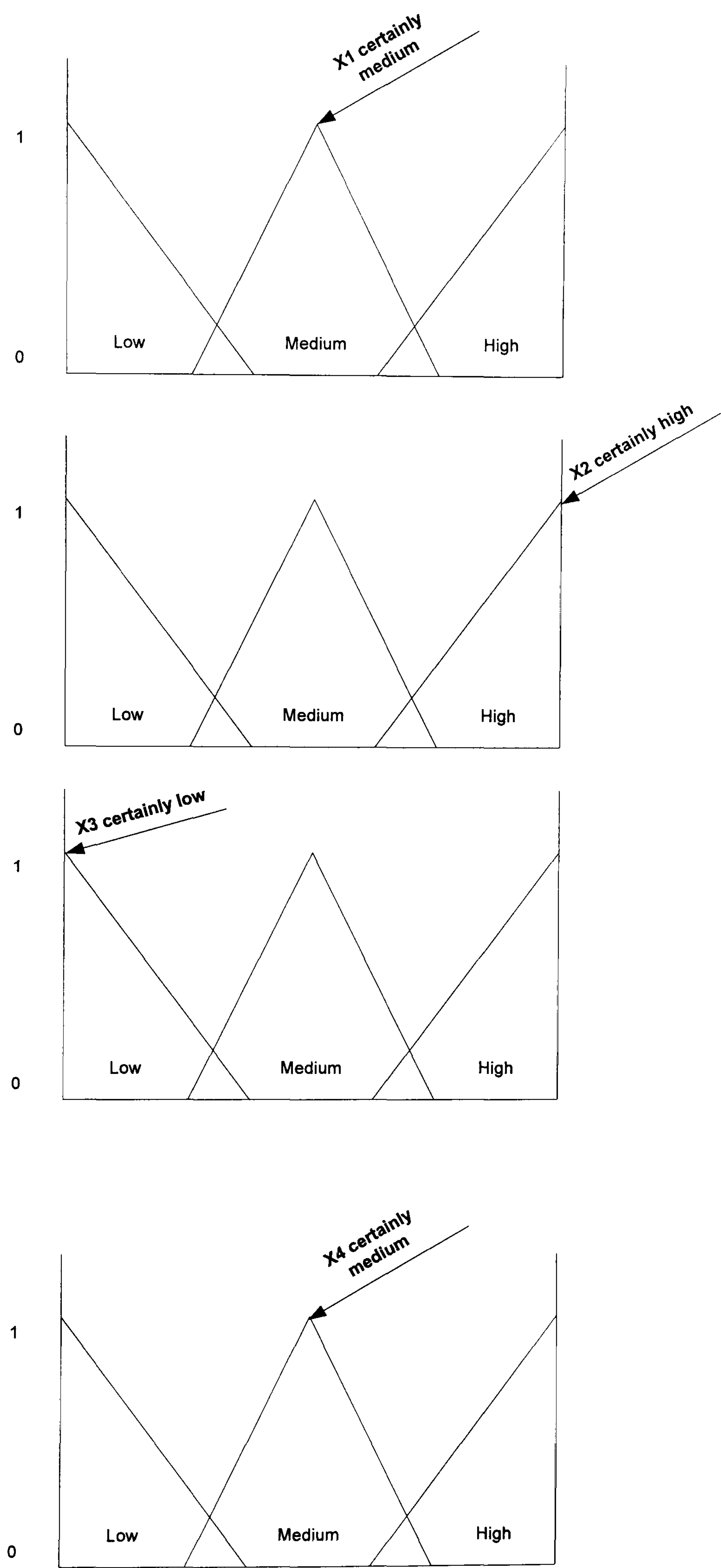




Figure A4.5: Choosing the Fuzzy Sets for Y in the Output in Developing Rules.

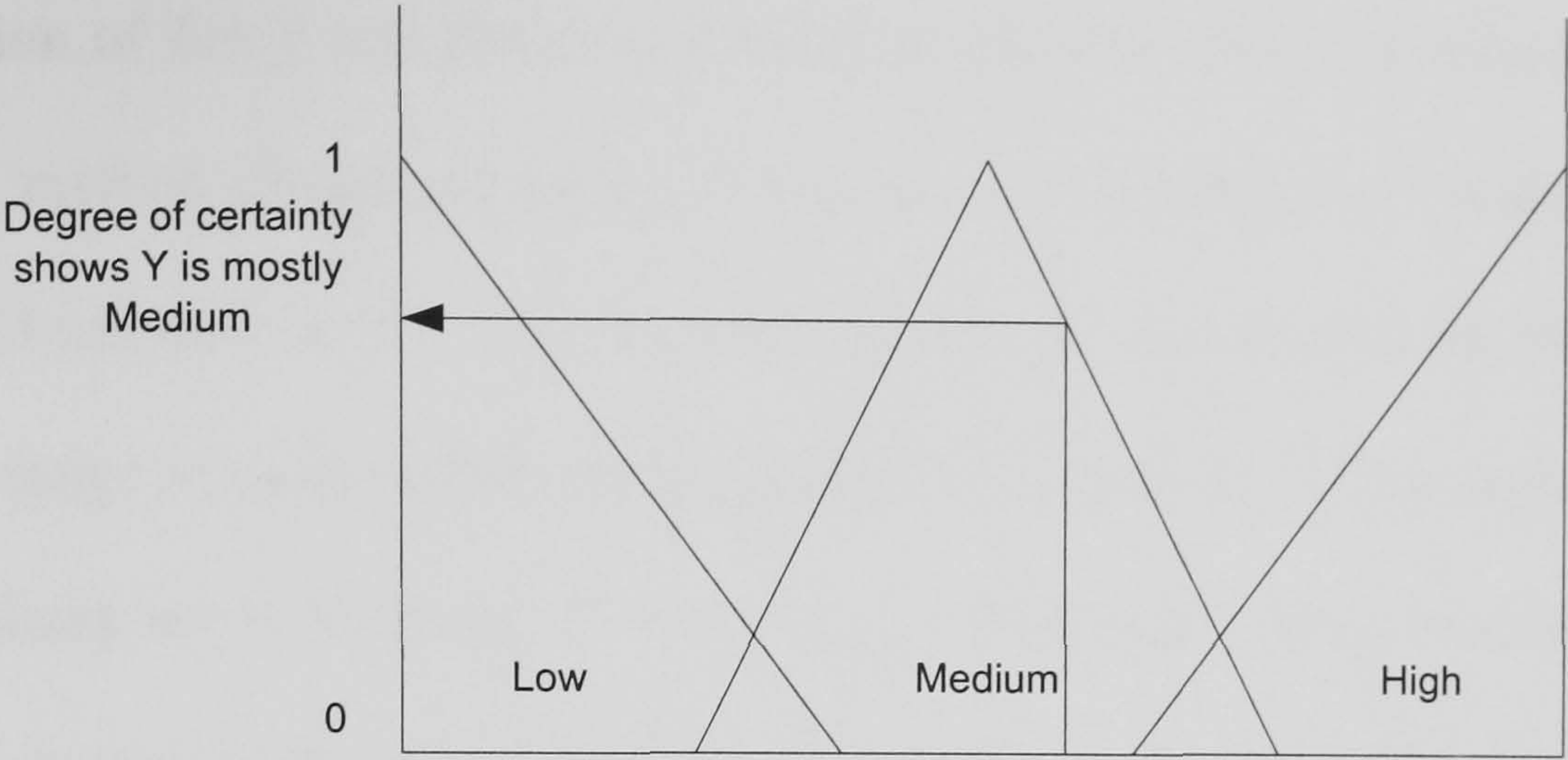


Figure A4.6: Systematically Building a Rule Base.

1.	If (X1 is 1) and (X2 is 1) and (X3 is 1) and (X4 is 1) then (Y is 1) (1)
2.	If (X1 is 1) and (X2 is 1) and (X3 is 1) and (X4 is 2) then (Y is 1) (1)
3.	If (X1 is 1) and (X2 is 1) and (X3 is 1) and (X4 is 3) then (Y is 1) (1)
4.	If (X1 is 1) and (X2 is 1) and (X3 is 1) and (X4 is 4) then (Y is 1) (1)
5.	If (X1 is 1) and (X2 is 1) and (X3 is 1) and (X4 is 5) then (Y is 1) (1)
6.	If (X1 is 1) and (X2 is 1) and (X3 is 2) and (X4 is 1) then (Y is 1) (1)
7.	If (X1 is 1) and (X2 is 1) and (X3 is 2) and (X4 is 2) then (Y is 1) (1)
8.	If (X1 is 1) and (X2 is 1) and (X3 is 2) and (X4 is 3) then (Y is 1) (1)
9.	If (X1 is 1) and (X2 is 1) and (X3 is 2) and (X4 is 4) then (Y is 1) (1)
10.	If (X1 is 1) and (X2 is 1) and (X3 is 2) and (X4 is 5) then (Y is 1) (1)
11.	If (X1 is 1) and (X2 is 1) and (X3 is 3) and (X4 is 1) then (Y is 1) (1)
12.	If (X1 is 1) and (X2 is 1) and (X3 is 3) and (X4 is 2) then (Y is 1) (1)
13.	If (X1 is 1) and (X2 is 1) and (X3 is 3) and (X4 is 3) then (Y is 1) (1)
14.	If (X1 is 1) and (X2 is 1) and (X3 is 3) and (X4 is 4) then (Y is 2) (1)
15.	If (X1 is 1) and (X2 is 1) and (X3 is 3) and (X4 is 5) then (Y is 2) (1)
16.	If (X1 is 1) and (X2 is 1) and (X3 is 4) and (X4 is 1) then (Y is 1) (1)
17.	If (X1 is 1) and (X2 is 1) and (X3 is 4) and (X4 is 2) then (Y is 1) (1)
18.	If (X1 is 1) and (X2 is 1) and (X3 is 4) and (X4 is 3) then (Y is 2) (1)
19.	If (X1 is 1) and (X2 is 1) and (X3 is 4) and (X4 is 4) then (Y is 2) (1)

Figure A4.7: Constructing the Rule-base.

if		and		and		and		Then
	X1 is		X2 is		X3 is		X4 is	Y is
	<div><div>1</div><div>2</div><div>3</div><div>4</div><div>5</div><div>none</div></div>		<div><div>1</div><div>2</div><div>3</div><div>4</div><div>5</div><div>none</div></div>		<div><div>1</div><div>2</div><div>3</div><div>4</div><div>5</div><div>none</div></div>		<div><div>1</div><div>2</div><div>3</div><div>4</div><div>5</div><div>none</div></div>	<div><div>1</div><div>2</div><div>3</div><div>4</div><div>5</div><div>none</div></div>
	<input type="checkbox"/> not		<input type="checkbox"/> not		<input type="checkbox"/> not		<input type="checkbox"/> not	<input type="checkbox"/> not



To form a complete rule base, a sample of which is shown in Figure A4.6, every combination of fuzzy sets from each input variable are chosen systematically. The particular method chosen for doing so was to keep the choice of fuzzy set for X1, X2 and X3 constant as ‘1’. ‘1’ is the term used in the experiments for the first fuzzy set in the range of each variable as highlighted in Figure A4.7. One rule was formed for each fuzzy set in X4 from ‘1’ to ‘5’ (e.g. if there are 5 fuzzy sets for each input variable), as seen in the first 5 rules in Figures A4.6 and A4.7. The fuzzy set for Y depends on the method shown by Figure A4.5. The process continues by choosing X1 as ‘1’, X2 as ‘1’, X3 now as ‘2’ and X4 is again worked through from ‘1’ to ‘5’. The process continues systematically until all combinations of all fuzzy sets from all inputs have been chosen. The process described is important for the factor, “percentage of the rule base” in section 4.6, “Description of the Experimental Trials”.

- (d) 350 test data points were randomly generated and input into the model. The individual percentage errors were averaged to calculate the Average Percentage Error (APE).

## **A4.2 Experimental Tables**

To allow brevity the tables for the experimental trials have been gathered below.



**Table A4.1: Design for Experimental Trials 1 (“The Basic Experiment”).**

<b>Experiment Number</b>	<b>Fuzzy Set Shape</b>	<b>Number of Fuzzy Sets per Variable</b>	<b>Overlap</b>	<b>Shoulder Width</b>	<b>% Number of Rules</b>	<b>Defuzzification Method</b>
1	Triangle	3	25	75	All	Centroid
2	Triangle	3	50	40	50	Bisector
3	Triangle	3	75	0	25	MOM
4	Triangle	5	25	75	50	MOM
5	Triangle	5	50	40	25	Centroid
6	Triangle	5	75	0	All	Bisector
7	Triangle	7	25	40	All	MOM
8	Triangle	7	50	0	50	Centroid
9	Triangle	7	75	75	25	Bisector
10	Trapezoidal	3	25	0	25	Centroid
11	Trapezoidal	3	50	75	All	Bisector
12	Trapezoidal	3	75	40	50	MOM
13	Trapezoidal	5	25	40	25	Bisector
14	Trapezoidal	5	50	0	All	MOM
15	Trapezoidal	5	75	75	50	Centroid
16	Trapezoidal	7	25	0	50	Bisector
17	Trapezoidal	7	50	75	25	MOM
18	Trapezoidal	7	75	40	All	Centroid



**Table A4.2: Alternative Defuzzification Methods.**

Experiment number	Fuzzy Set Shape	Number of Fuzzy Sets	% Overlap	% Shoulder Width	% Number of Rules	Defuzzification Methods
1	Triangle	3	25	75	All	LOM
2	Triangle	3	50	40	50	SOM
3	Triangle	3	75	0	25	MOM
4	Triangle	5	25	75	50	MOM
5	Triangle	5	50	40	25	LOM
6	Triangle	5	75	0	All	SOM
7	Triangle	7	25	40	All	MOM
8	Triangle	7	50	0	50	LOM
9	Triangle	7	75	75	25	SOM
10	Trapezoidal	3	25	0	25	LOM
11	Trapezoidal	3	50	75	All	SOM
12	Trapezoidal	3	75	40	50	MOM
13	Trapezoidal	5	25	40	25	SOM
14	Trapezoidal	5	50	0	All	MOM
15	Trapezoidal	5	75	75	50	LOM
16	Trapezoidal	7	25	0	50	SOM
17	Trapezoidal	7	50	75	25	MOM
18	Trapezoidal	7	75	40	All	LOM

**Table A4.3: Gaussian Membership Functions.**

Experiment Numbers	Number of Fuzzy Sets per Variable	% Standard Deviation	Defuzzification Methods	Blank
1	3	25	Centroid	
2	3	50	Bisector	
3	3	75	MOM	
4	5	25	Bisector	
5	5	50	MOM	
6	5	75	Centroid	
7	7	25	MOM	
8	7	50	Centroid	
9	7	75	Bisector	



**Table A4.4: “Gaussian Membership Functions” and “Changing the Defuzzification Methods”.**

Experiment numbers	Number of Fuzzy Sets	%Standard Deviation	Defuzzification Methods	Blank
1	3	25	LOM	
2	3	50	SOM	
3	3	75	MOM	
4	5	25	SOM	
5	5	50	MOM	
6	5	75	LOM	
7	7	25	MOM	
8	7	50	LOM	
9	7	75	SOM	

**Table A4.5: Taguchi Orthogonal Array for Experimental Trials 7.**

	Fuzzy Sets (A=2,B=3,C=4,D=2)	Epoch (A=50, B=60, C=70, D=80)	Data Points (A=50, B=300, C=500, D=750)	Column 4 (not used)	Y(Not used)
1	A	A	A	A	A
2	A	B	B	B	B
3	A	C	C	C	C
4	A	D	D	D	D
5	B	A	B	C	D
6	B	B	A	D	C
7	B	C	D	A	B
8	B	D	C	B	A
9	C	A	C	D	B
10	C	B	D	C	A
11	C	C	A	B	D
12	C	D	B	A	C
13	D	A	D	B	C
14	D	B	C	A	D
15	D	C	B	D	A
16	D	D	A	C	B



**Table A4.6: An L16 for Subtractive Clustering.**

	X1	X2	X3	X4	Y	
1	A	A	A	A	A	
2	A	B	B	B	B	A=0.2
3	A	C	C	C	C	B=0.3
4	A	D	D	D	D	C=0.5
5	B	A	B	C	D	D=0.75
6	B	B	A	D	C	
7	B	C	D	A	B	
8	B	D	C	B	A	
9	C	A	C	D	B	
10	C	B	D	C	A	
11	C	C	A	B	D	
12	C	D	B	A	C	
13	D	A	D	B	C	
14	D	B	C	A	D	
15	D	C	B	D	A	
16	D	D	A	C	B	



**Table A4.7: Taguchi L16 Orthogonal Array Used With The Best Structure From  
The Mamdani Method.**

Experiment Number	X1	X2	X3	X4	Y (Column left blank)	
1	A	A	A	A		
2	A	B	B	B		A=3
3	A	C	C	C		B=9
4	A	D	D	D		C=27
5	B	A	B	C		D=81
6	B	B	A	D		
7	B	C	D	A		
8	B	D	C	B		
9	C	A	C	D		
10	C	B	D	C		
11	C	C	A	B		
12	C	D	B	A		
13	D	A	D	B		
14	D	B	C	A		
15	D	C	B	D		
16	D	D	A	C		



Table A4.8: The  $L_{16}$  Table Used to Identify Interactions. Column Numbers are Shown at the Top of Each Column.

Experiment Number	2 (A)	4 (B)	8 C	15 (D)	1 (Y)	A*B(6)	C*D(7)	A*C(10)	B*D(11)	B*C(12)	A*D(13)
1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	2	2	1	1	1	2	2	2	2
3	1	2	1	2	1	2	2	1	1	2	2
4	1	2	2	1	1	2	2	2	2	1	1
5	2	1	1	2	1	2	2	2	2	1	1
6	2	1	2	1	1	2	2	1	1	2	2
7	2	2	1	1	1	1	1	2	2	2	2
8	2	2	2	2	1	1	1	1	1	1	1
9	1	1	1	2	2	1	2	1	2	1	2
10	1	1	2	1	2	1	2	2	1	2	1
11	1	2	1	1	2	2	1	1	2	2	1
12	1	2	2	2	2	2	1	2	1	1	2
13	2	1	1	1	2	2	1	2	1	1	2
14	2	1	2	2	2	2	1	1	2	2	1
15	2	2	1	2	2	1	2	2	1	2	1
16	2	2	2	1	2	1	2	1	2	1	2



**APPENDIX FOR CHAPTER 5, A5**

**A5.1 Numerical Results Tables**

The results for Chapter five are summarised in the following tables, graphs, and bar charts.

**Table A5.1: Experimental Trials 1. Results of Individual Trials.**

<b>Cost Model Number</b>	<b>MAPE</b>
1	434.5
2	625.4
3	562.1
4	375.5
5	700.2
6	256.2
7	86.2
8	189
9	347.7
10	1077.6
11	535.5
12	425.6
13	1001.4
14	141.5
15	317.1
16	488.4
17	479.1
18	164.4
<b>Best and Worst Structures Found in Table 5.4</b>	



**Table A5.2: Experimental Trials 1: Effect of Individual Structural Elements.**

Fuzzy Logic Structural Elements	Fuzzy Logic Structural Element Factors	Cost Model Numbers	MAPE
Number of Membership Functions	3	1,2,3,10,11,12	610.1
	5	4,5,6,13,14,15	465.3
	7	7,8,9,16,17,18	292.5
Percentage Overlap	25	1,4,7,10,13,16	577.3
	50	2,5,8,11,14,17	445.1
	75	3,6,9,12,15,18	345.5
Percentage Shoulder Width	0	3,6,8,10,14,16	452.5
	40	2,5,7,12,13,18	500.5
	75	1,4,9,11,15,17	414.9
Percentage Number of Rules	100	1,6,7,11,14,18	269.7
	50	2,4,8,12,15,16	403.5
	25	3,5,9,10,13,17	694.7
Defuzzification Methods	Centroid	1,5,8,10,15,18	480.5
	Bisector	2,6,9,11,13,16	542.4
	Mean of Maximum	3,4,7,12,14,17	345

**Table A5.3: The Numerical Effect of Individual Structural Elements.**

Structural Element	Effect of Structural Element
e.g. Number of Fuzzy Sets	e.g. (difference of highest average MAPE and lowest average MAPE), i.e. (292.5-610.1), i.e. -317.6
Percentage Overlap	(345.5-577.3), i.e. -231.8
Percentage Shoulder Width	(414.9-452.5), i.e. -37.6
Percentage Number of Rules	(269.7-694.7), i.e. -425.0
Defuzzification Methods	(345.0-542.4), i.e. -197.4



**Table A5.4: Experimental Trials 1. Effect of Changing Random Number Sets**

Random Number Set	Best Set of Structural Elements	Best Set of Structural Elements APE	Worst Set of Structural Elements	Worst Set of Structural Elements APE	Overall Average	Best Model	Worst Model
1	7 fuzzy sets, 75% overlap, 0% shoulder width, 100% rules, MOM	99.7	3 fuzzy sets, 25% overlap, 75% shoulder width, 25% number of rules, bisector	931.3	456	86.2	1077.6
2	7 fuzzy sets, 75% overlap, 0% shoulder width, 100% rules, MOM	93.7	3 fuzzy sets, 25% overlap, 75% shoulder width, 25% number of rules, bisector	982.2	449.3	79.5	1107.4



Figure A5.1: Experimental Trials 1: Relative Effect of Structural Elements.

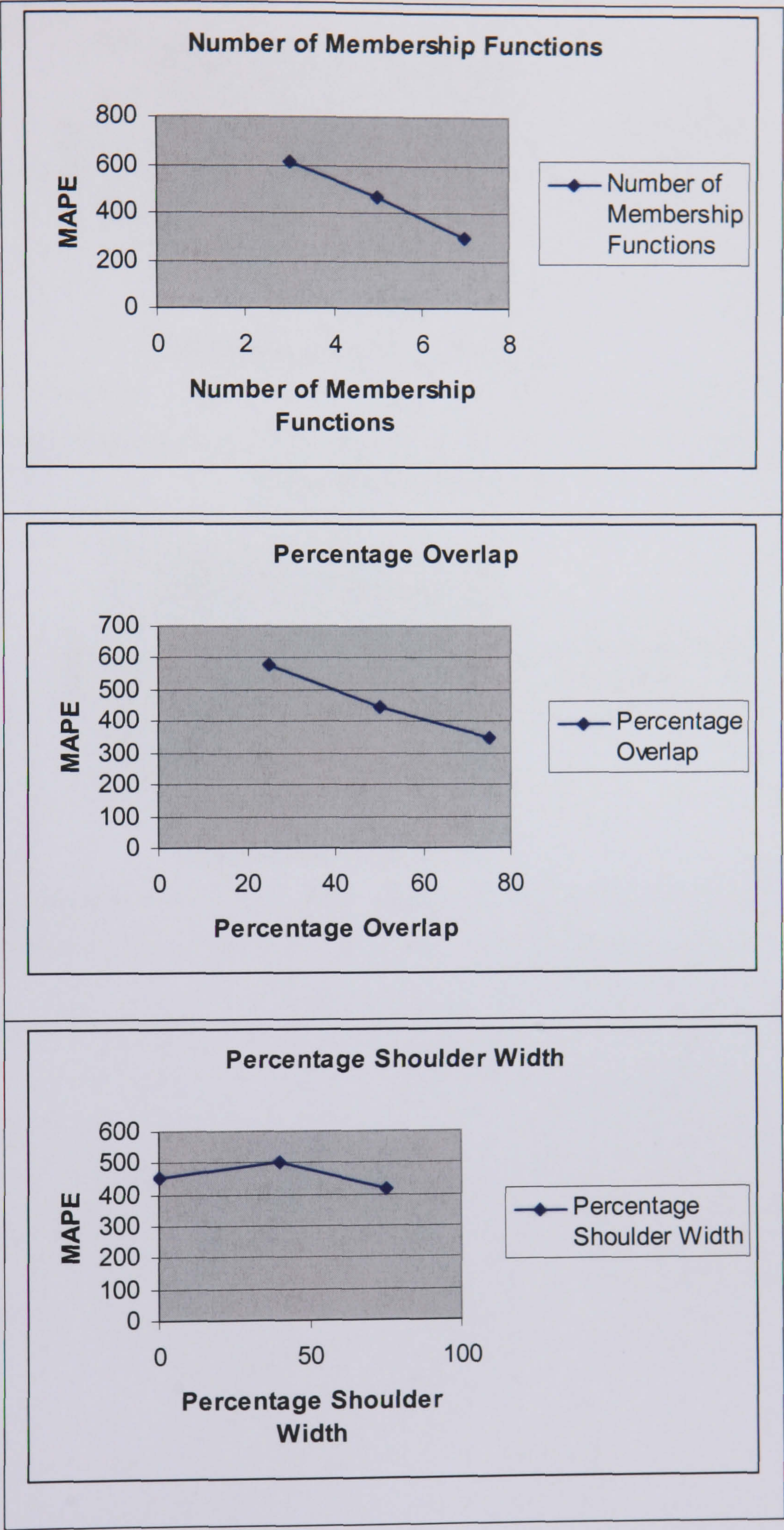
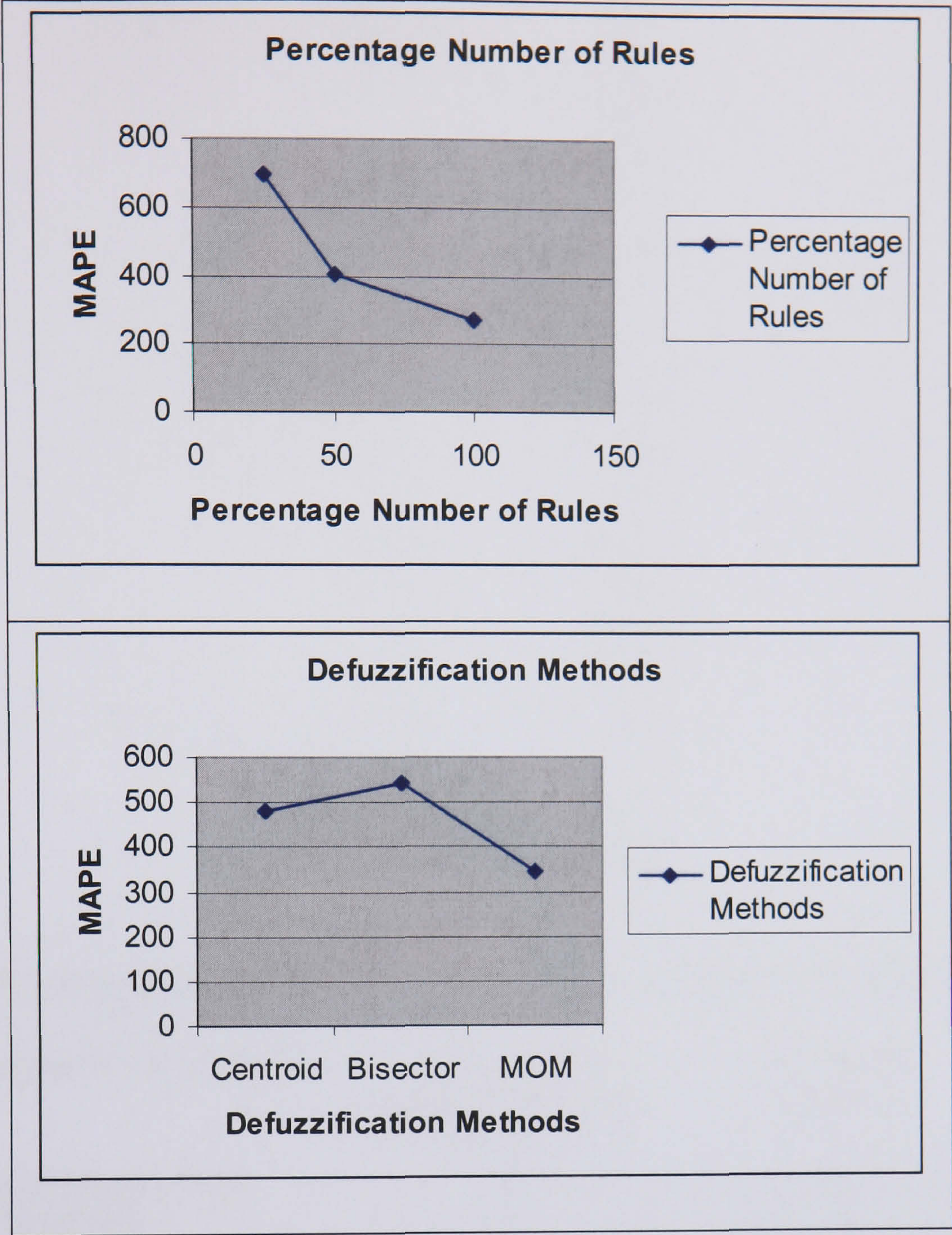




Figure A5.1: Continued





**Table A5.5: Experimental Trials 2: Individual Trials.**

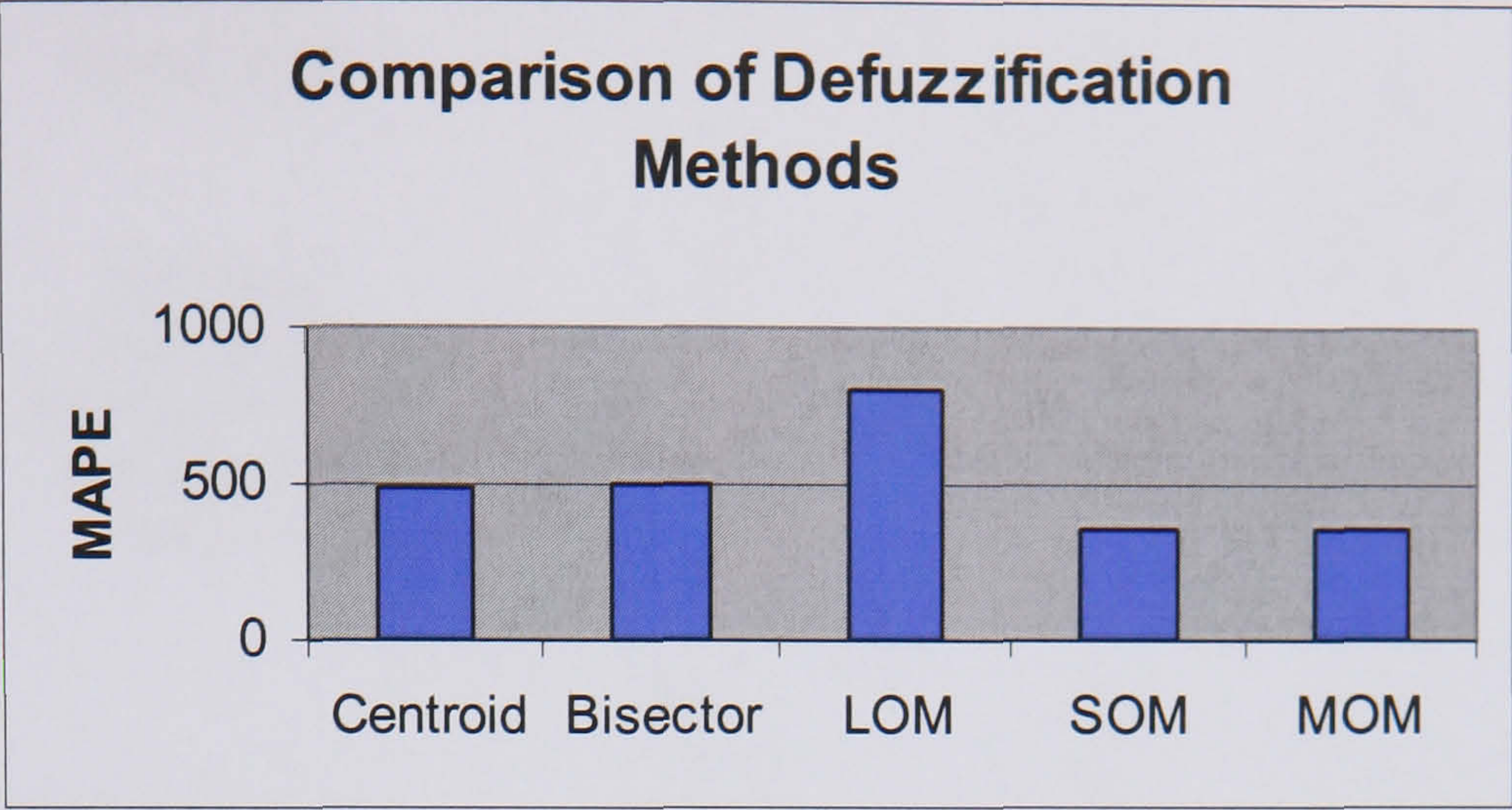
<b>Cost Model Number</b>	<b>APE</b>
1	1707.9
2	157.4
3	522.1
4	418.2
5	753.8
6	64.6
7	79.5
8	251.6
9	144.1
10	1246
11	80.5
12	450.9
13	1193.2
14	166.4
15	627.9
16	461.8
17	505.1
18	217.3
Best structure	54.1
Worst structure	1251.6

**Table A5.6: Experimental Trials 2: Effects of Individual Structural Elements.**

<b>Fuzzy Logic Structural Elements</b>	<b>Fuzzy Logic Structural Element Factors</b>	<b>Cost Model Numbers</b>	<b>MAPE</b>
<b>Number of Membership Functions</b>	3	1,2,3,10,11,12	694.1
	5	4,5,6,13,14,15	537.3
	7	7,8,9,16,17,18	276.6
<b>Percentage Overlap</b>	25	1,4,7,10,13,16	851.1
	50	2,5,8,11,14,17	319.1
	75	3,6,9,12,15,18	337.8
<b>Percentage Shoulder Width</b>	0	3,6,8,10,14,16	452.1
	40	2,5,7,12,13,18	475.3
	75	1,4,9,11,15,17	580.6
<b>Percentage Number of Rules</b>	100	1,6,7,11,14,18	386
	50	2,4,8,12,15,16	394.6
	25	3,5,9,10,13,17	727.4
<b>Defuzzification Methods</b>	Largest of Maximum	1,5,8,10,15,18	800.7
	Smallest of Maximum	2,6,9,11,13,16	350.3
	Mean of Maximum	3,4,7,12,14,17	357



**Figure A5.2: Comparison of Defuzzification Methods From Experimental Trials 1 (Centroid, Bisector) and 2 (LOM, SOM, MOM).**



**Table A5.7: Experimental Trials 3. Individual Trials.**

Cost Model Number	APE, %
1	847.0
2	1196.0
3	281.8
4	783.9
5	162.5
6	1547.6
7	88.7
8	1636.1
9	1659.0
Best Structure	349.5
Worst Structure	1623.3

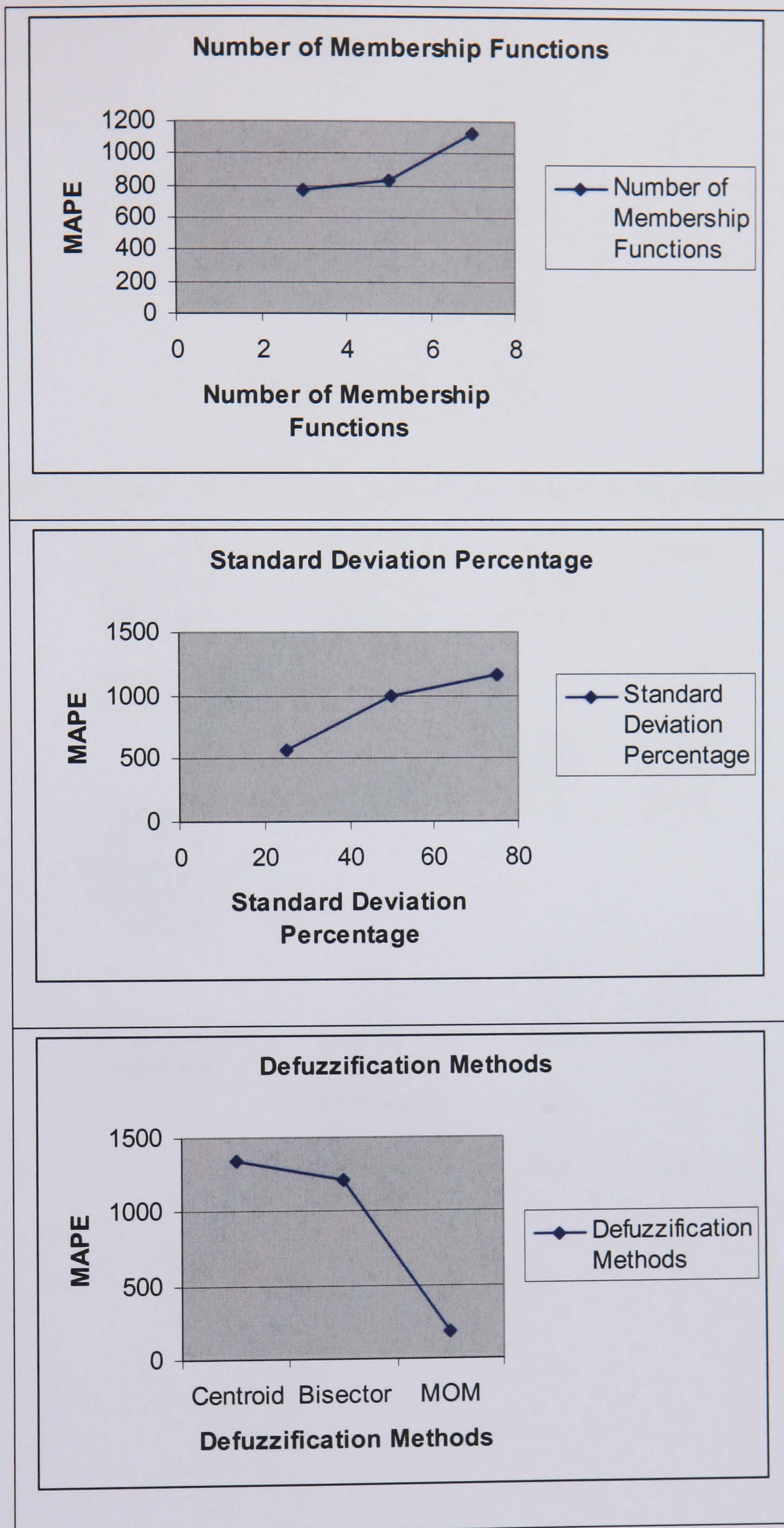


**Table A5.8: Experimental Trials 3: Effects of Individual Structural Elements.**

Fuzzy Logic Structural Elements	Fuzzy Logic Structural Element Factors	Model Numbers	MAPE
Fuzzy Sets	3	1,2,3	774.9
	5	4,5,6	831.3
	7	7,8,9	1127.9
	25	1,4,7	573.2
Standard Deviation: Percentage of the Variable Range			
	50	2,5,8	998.2
	75	3,6,9	1162.8
	Centroid	1,6,8	1343.6
	Bisector	2,4,9	1213.0
	Mean of Maximum	3,5,7	177.7
Defuzzification Methods			



**Figure A5.3: Experimental Trials 3: Effect of Fuzzy Logic Structural Elements.**





**Table A5.9: Experimental Trials 4: Results of Individual Trials.**

Cost Model Number	APE
1	580.6
2	77.8
3	281.8
4	61.7
5	162.5
6	321.1
7	88.7
8	176.8
9	56.5
Best Structure	58.9
Worst Structure	787.7

**Table A5.10: Experimental Trials 4: Effect of Individual Structural Elements.**

Fuzzy Logic structural Elements	Fuzzy Logic Structural Element factors	Cost Model Numbers	MAPE
Number of Fuzzy Sets	3	1,2,3	313.4
	5	4,5,6	181.8
	7	7,8,9	107.3
Standard Deviation: Percentage of the Variable Range	25	1,4,7	243.7
	50	2,5,8	139.1
	75	3,6,9	219.8
Defuzzification Methods	Largest of Maximum	1,6,8	359.5
	Smallest of Maximum	2,4,9	65.4
	Mean of Maxima	3,5,7	177.7



Figure A5.4: Experimental Trials 3 and 4: Defuzzification Methods.

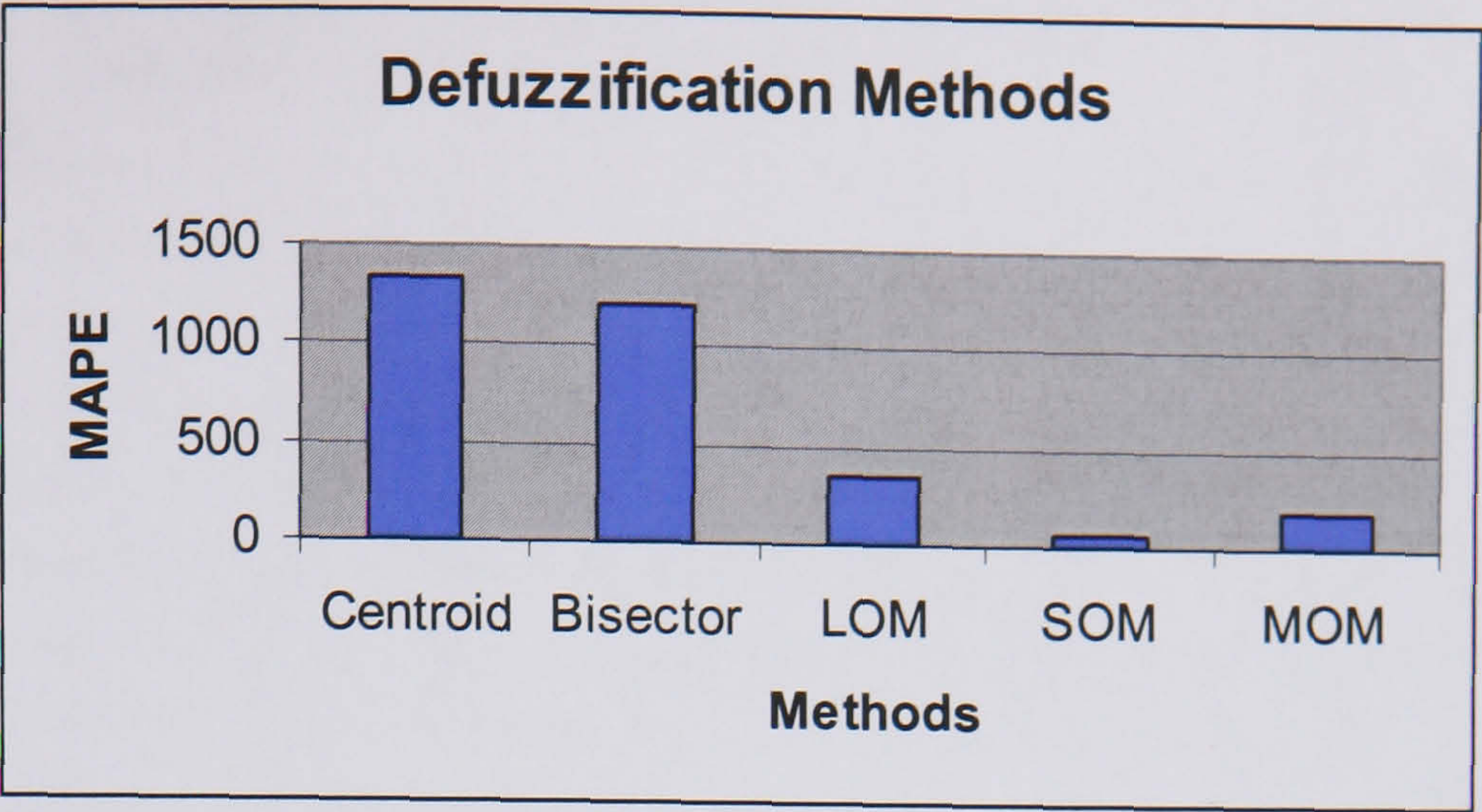


Table A5.11: Use of the Product Rule in Experimental Trials 1-4

	Best Structure	Best Model	Best Structure	Best Model
AND/implication	Exp Trial 1	Exp Trial 1	Exp Trial 2	Exp Trial 2
min/prod MAPE	60.5	60.7	60.2	70.4
min/prod sd	36.3	75.6	36.60	38.2
prod/prod MAPE	60.0	54.7	54.0	71.06
prod/prod sd	34.7	57.2	37.4	39.8
Prod/min MAPE	145.8	88.7	65.0	67.8
Prod/min sd	234.4	121.6	27.7	27.5

Table A5.11: Continued

	Best Structure	Best Model	Best Structure	Best Model
AND/implication	Exp Trial 3	Exp Trial 3	Exp Trial 4	Exp Trial 4
min/prod MAPE	48.2	56.0	59.0	59.0
min/prod sd	38.5	34.4	35.1	34.4
prod/prod MAPE	54.1	59.1	58.8	57.9
prod/prod sd	46.4	34.5	33.8	34.50
Prod/min MAPE	247.7	138.8	63.7	61.5
Prod/min sd	456.2	217.9	30.7	30.7



Table A5.12: Adaptive Neuro-Fuzzy Inference System

	Fuzzy Sets (2,3,4,5)	Epoch (50,60,70,80)	Data Points (50,300,500,750)	X4	Y(Not used)	APE	APSD
1	A	A	A	A	A	428.3	860.3
2	A	B	B	B	B	18.7	38.5
3	A	C	C	C	C	10.8	24.3
4	A	D	D	D	D	12.6	28.4
5	B	A	B	C	D	281.4	729.9
6	B	B	A	D	C	124.2	241.4
7	B	C	D	A	B	52.3	145.9
8	B	D	C	B	A	508.4	3414.5
9	C	A	C	D	B	4041.2	11235
10	C	B	D	C	A	1644.4	7399.2
11	C	C	A	B	D	97.3	174.5
12	C	D	B	A	C	2819	12519
13	D	A	D	B	C	22902	262520
14	D	B	C	A	D	974.9	2947
15	D	C	B	D	A	2360	11790
16	D	D	A	C	B	131.6	212.9

Table A5.13: Experimental Trials 7. Effect of Individual Structural Elements.

Number of Fuzzy Sets		
2	1,2,3,4	117.6
3	5,6,7,8	241.6
4	9,10,11,12	2150.5
2	13,14,15,16	6592.1
Number of Epochs		
50	1,5,9,13	6913.2
60	2,6,10,14	690.6
70	3,7,11,15	630.1
80	4,8,12,16	3471.6
Number of Data Points		
50	1,6,11,16	195.4
300	2,5,12,15	1369.8
500	3,8,9,14	1383.8
750	4,7,10,13	6152.8



**Table A5.14: The Effect of Varying the Number of Data Points for the Best Model**

**/ Best Structure**

<b>Data Points</b>	<b>Average Percentage Error</b>	<b>Average Percentage Standard Deviation (APSD)</b>
10	146.7	341.8
20	1890.4	5938.5
30	976.7	1927.0
40	900.8	1868.3
50	467.8	931.3
100	57.7	120.8
150	41.6	104.4
200	17.2	56.7
250	26.9	76.4
300	3.1	8.5
350	12.3	28.8
400	12.0	21.5
450	11.2	23.7
500	16.2	32.7
550	8.7	18.9
600	9.7	20.2
650	9.4	19.9
700	1.6	3.2
750	8.1	16.9



**Tables A5.15: Results of Individual Clustering Trials.**

<b>Cost Model Number</b>	<b>APE</b>
1	2103.7
2	666.1
3	36.7
4	73.1
5	48.1
6	16.3
7	73.8
8	23.2
9	36.6
10	39.9
11	9.9
12	10.3
13	63.1
14	28.3
15	13.6
16	5

<b>Cost Model Number</b>	<b>APSD</b>
1	15635.0
2	23864.0
3	74.9
4	225.4
5	287.0
6	31.4
7	92.4
8	89.7
9	61.7
10	83.6
11	12.9
12	12.8
13	111.0
14	74.3
15	22.4
16	8.1



Table A5.16: Effect of Individual Structural Elements

Variables	Cluster Influence	Cost Model Numbers	MAPE
X1	A	1,2,3,4	719.9
	B	6,5,7,8	40.3
	C	9,10,11,12	24.2
	D	13,14,15,16	27.5
X2	A	1,5,9,13	562.9
	B	2,6,10,14	187.6
	C	3,7,11,15	33.5
	D	4,8,12,16	27.9
X3	A	1,6,11,16	533.7
	B	2,5,12,15	184.5
	C	3,8,9,14	31.2
	D	4,7,10,13	62.5
X4	A	1,7,12,14	554.0
	B	2,8,11,13	190.6
	C	3,5,10,16	32.4
	D	4,6,9,15	34.9
Y	A	1,8,10,15	545.1
	B	2,7,9,16	195.4
	C	3,6,12,13	31.6
	D	4,5,11,14	39.9

Table A5.17: Comparison of Random Number Sets

Random Number Set	Best Set of Structural Elements	Best Set of Structural Elements MAPE	Worst Set of Structural Elements	Worst Set of Structural Elements MAPE	Overall Average	Best Model	Worst Model
1	C,D,C,C,C	30.3	A,A,A,A,A	977.8	203.0	5.0	2103.7
2	C,C,C,D,C	27.3	A,A,A,A,A	2637	223.6	4.7	2745.1



**Table A5.18: Individual Trials Testing the Best Model from Table 5.9 in  
Estimating Linear Models Chosen Using the Orthogonal Array in Table 5.19**

<b>Cost Model Number</b>	<b>Average Percentage Error (APE)</b>
1	15.2
2	13.9
3	11.9
4	13.6
5	14.9
6	14.8
7	13.0
8	13.1
9	15.9
10	15.6
11	14.9
12	12.6
13	15.7
14	12.1
15	14.6
16	13.2



**Table A5.19: Taguchi Orthogonal Array to Choose the Linear Models for Which  
to Test the Best Model from Table A5.9.**

Experiment Number	X1	X2	X3	X4	Y		Average Percentage Error (APE)	Average Percentage Standard Deviation (APSD)
1	A	A	A	A			15.2	22.0
2	A	B	B	B		A=3	13.9	22.2
3	A	C	C	C		B=9	11.9	19.7
4	A	D	D	D		C=27	13.6	20.5
5	B	A	B	C		D=81	14.9	23.1
6	B	B	A	D			14.8	23.0
7	B	C	D	A			13.0	19.1
8	B	D	C	B			13.1	19.7
9	C	A	C	D			15.9	25.1
10	C	B	D	C			15.6	26.0
11	C	C	A	B			14.9	25.6
12	C	D	B	A			12.6	17.8
13	D	A	D	B			15.7	23.4
14	D	B	C	A			12.1	19.3
15	D	C	B	D			14.6	23.3
16	D	D	A	C			13.2	20.8



**Table A5.20: The use of MLR Method to Estimate the Linear Model Equation 4.3,  
and Varying the Number of Data Points.**

Number of data points	Average Percentage Error (APE)
5	0
10	0
20	0
30	0
40	0
50	0
60	0
70	0
80	0
90	0
100	0
500	0
1000	0

**Table A5.21: Experimental Trials 11: Investigating Interaction Effects for  
Subtractive Clustering**

Experiment Number	APE
1	631.6
2	88.3
3	7.6
4	61.5
5	9.6
6	68.7
7	4.4
8	43.0
9	394.7
10	493.3
11	167.3
12	76.3
13	448.5
14	58.9
15	6.4
16	33.9



**Table A5.22: The Average MAPE for Cluster Influence for Variables X1, X2, X3 and X4 and Different Levels of Interactions (X1\*X2), (X1\*X3), (X1\*X4), (X2\*X3), (X2\*X4), (X3\*X4).**

Variables	Cluster Influence	Cost Model Numbers	Average MAPE
X1	A	1,2,3,4,9,10,11,12	240.1
	B	5,6,7,8,13,14,15,16	84.2
X2	A	1,2,5,6,9,10,13,14	274.2
	B	3,4,7,8,11,12,15,16	50.1
X3	A	1,3,5,7,9,11,13,15	208.8
	B	2,4,6,8,10,12,14,16	115.5
X4	A	1,4,6,7,10,11,13,16	238.7
	B	2,3,5,8,9,12,14,15	85.6
Y	A	1,2,3,4,5,6,7,8	114.3
	B	9,10,11,12,13,14,15,16	209.9
X1*X2	A	1,2,7,8,9,10,15,16	212.0
	B	3,4,5,6,11,12,13,14	112.3
X1*X3	A	1,3,6,8,9,11,14,16	175.7
	B	2,4,5,7,10,12,13,15	148.5
X1*X4	A	1,4,5,8,10,11,14,15	184.0
	B	2,3,6,7,9,12,13,16	140.3
X2*X3	A	1,4,5,8,9,12,13,16	212.4
	B	2,3,6,7,10,11,14,15	111.9
X3*X4	A	1,2,7,8,11,12,13,14	189.8
	B	3,4,5,6,9,10,15,16	134.5



**Table A5.23. Repeat Experiments with Absolute Instead of Percentage Errors.**

<b>Experimental Trials / Model</b>	<b>Average Absolute Error (AAE)</b>	<b>Average Absolute Standard Deviation (AASD)</b>	<b>Output Variable Range</b>	<b>AAE as a percentage of the output variable range</b>	<b>AASD as a percentage of the output variable range</b>
Experimental Trials 1 / “best structure”	15098	13086	1020 to 367,697	4.12	3.57
Experimental Trials 4 / “best model”	15376	17237	1020 to 367,697	4.19	4.70
Experimental Trials 8 / “best model”	719	836	1020 to 367,697	0.20	0.23
Experimental Trials 8 / “best model”	1082	1563	1020 to 367,697	0.30	0.43
Experimental Trials 10, testing the “best model” from Experimental Trials 4 on a linear model, Experiment number 3	114,210	136120	29,705 to 2,727,039	4.23	5.05



## APPENDIX FOR CHAPTER SIX, A6

### A6.1 Trends Obtained from the Taguchi Methodology as a Systematic Method

Experimental trials 1, show that *assuming weak interactions* (i.e. each MAPE can be affected by 20% because of interactions), then *MAPE is improved by*

- (1) increasing the number of fuzzy sets,
- (2) increasing the percentage of rules used as chosen in this case (Section A4.1),
- (3) increasing overlap,
- (4) choosing percentage shoulder width to be 75% in this case (no clear trend existed), and
- (5) choosing the maximum based defuzzification method.

Experimental trials 2 show that *assuming weak interactions* (i.e. each MAPE can be affected by 20% because of interactions), then *MAPE is improved by*

1. increasing the number of fuzzy sets,
2. increasing the percentage of rules used from 25% to either 50% or 100%,
3. increasing overlap,
4. increasing percentage shoulder width, and
5. choosing the smallest or mean of maximum method for defuzzification over the largest of maximum method.

Experimental trials 3 show that *assuming weak interactions* (i.e. each MAPE can be affected by 20% because of interactions), then *MAPE is improved by*

1. decreasing the number of fuzzy sets,
2. increasing the standard deviation of the Gaussian fuzzy sets, and



3. the Mean of Maximum method performed the best followed by the Bisector method and the Centroid method.

Experimental trials 4 show that *assuming weak interactions* (i.e. each MAPE can be affected by 20% because of interactions), then *MAPE is improved by*

1. increasing the number of fuzzy sets,
2. no clear trend in standard deviation but the 50% of the variable range as standard deviation performed the best, and
3. the smallest of maximum method performed the best followed by the mean of maximum and the largest of maximum.

Experimental Trials 5 showed that increasing the number of output fuzzy sets had no clear effect on APE.

Experimental Trials 6 showed that the ANFIS model used, outperformed multiple linear regression analysis over a range of data points, i.e. for 300 to 750 for the APE error measure. The standard deviation of the error notably was better than the MLR method towards the higher range of the number of data points above but was very much worse to comparable over the whole range at *key points*.

The introduction of the product operator with the minimum operator for AND or implication or both being product, improved the APE.

Experimental Trials 7 showed that the level 0 method, ANFIS could produce a model of 1.6 APE with only 2 fuzzy sets per variable, i.e. in this case 16 rules, and an APSD of



3.2. ANFIS models with a large number of parameters took of the order of days to train the specified number of epochs, and frequently had poor errors (Table A5.12). It was thought that the poor errors were due to the incomplete training of the network.

Experimental Trials 8 showed that a general trend of increasing accuracy with higher cluster influence values occurred but that sometimes the influence due to C (0.5) produced a better accuracy than the higher D (0.75). Interactions, as shown in Experimental Trials 11, were a possible explanation.

### **A6.2 Problems with Systematic Method: What Interactions?**

Figure 6.1 shows an interaction that explains the presence of a “best model” and a “best structure” in Experimental Trials 1 and 2. The two factors of “percentage shoulder width” and “overall percentage overlap” interact to change the shape of a fuzzy set. In addition, it is noted that “overall percentage overlap” and “number of fuzzy sets” interact to determine how many units overlap there actually are between fuzzy sets. The change in shape means that an input into the model can have different membership values when the shape changes. Figure A6.1 shows that the interaction between 40% shoulder width and overall percentage overlap changes the degree of membership of the input to the set from 0.3 to 0.7 when overall percentage overlap increases (the numbers are for demonstration purposes only).



**Figure A6.1: Explaining the Presence of Interactions**

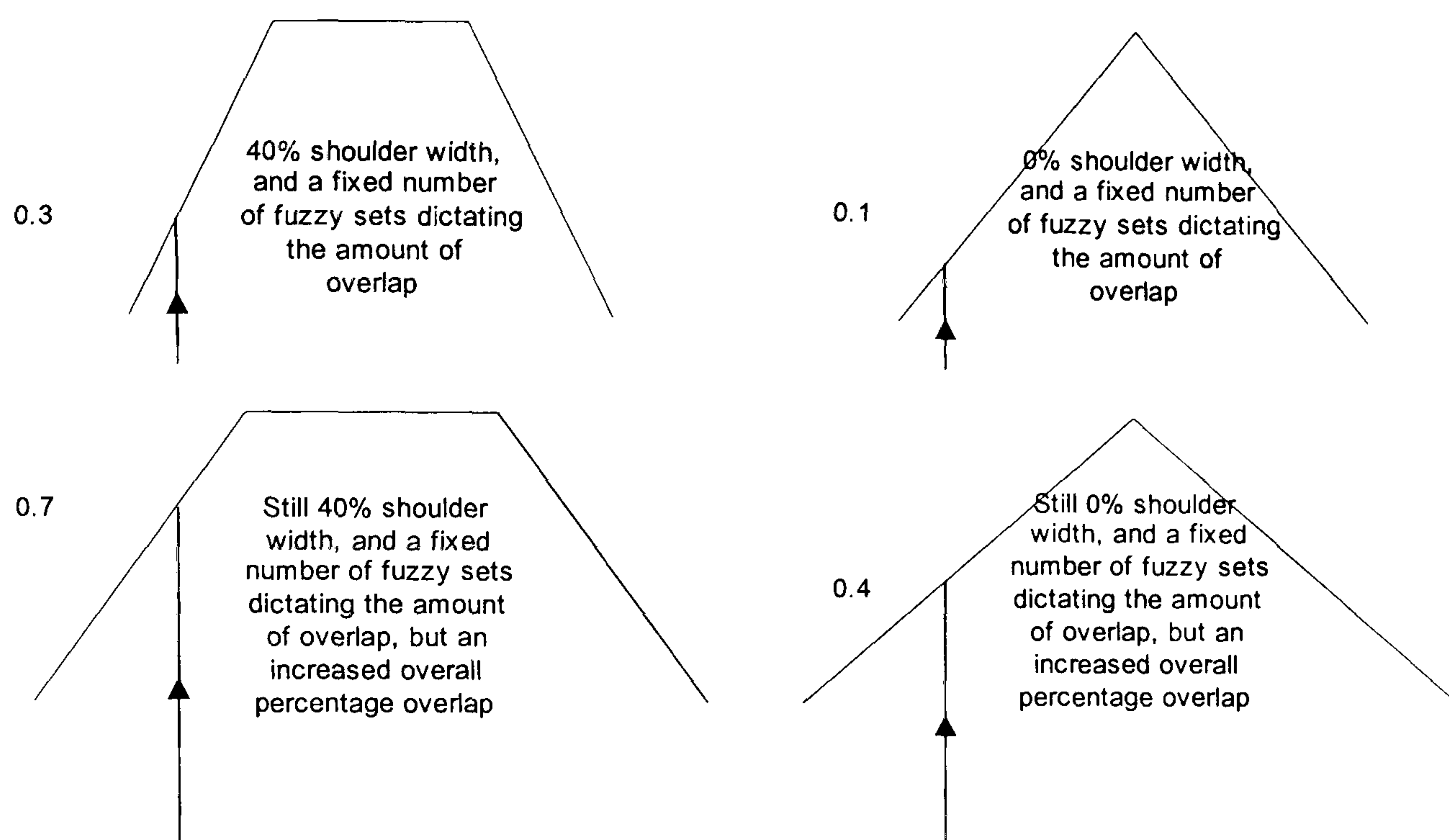
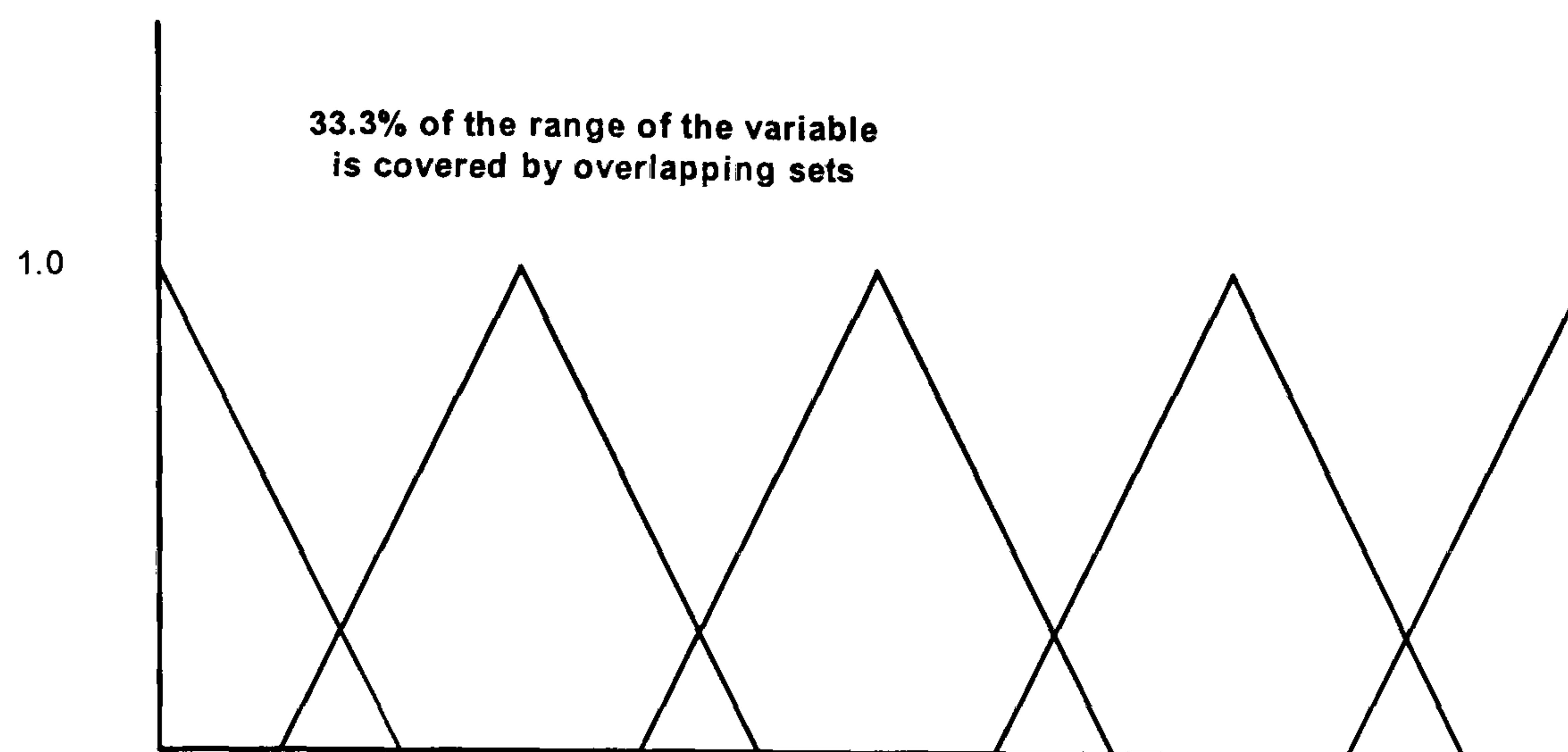


Figure A6.2 helps to explain how fuzzy sets interact with the notion of overlap. This research constrains the overlap to be measured as a percentage of the range of the variable. In effect this means that more fuzzy sets covering this range, in turn, means that the fixed amount of overlap has to be shared between more overlaps. This effect is that the slope of the triangles and trapezoids is constrained to be steeper if the shoulder width is kept constant. The effect of changing the shoulder width leads to the already explained interaction in Figure A6.1.



**Figure A6.2: Overlap is Spread Over the Range**



Further interactions occur between fuzzy sets, defuzzification methods, and the radii of influence within clustering. For example it is clear that changing the shape of fuzzy sets from symmetrical to unsymmetrical fuzzy sets, shall interact with the centre of area defuzzification method. The interactions between the radii of influence assigned to each individual variable for the subtractive clustering based method in Equation (4.1) are shown by the non-parallel nature of the lines in figures 5.10. Importantly it is noted that interactions due to the structure of Equation (4.1) are considered, but that lots of the interactions cannot be explained by this. It is apparent that the fuzzy logic method should be thought of as a complete system of structural elements rather than as individual parts (See A1.10).

### **A6.3 Observations of the Numerical Results**

Observation of the results from Chapter 5 can be summarised by:

- (1) models using Gaussian fuzzy sets produced the least APE,
- (2) models using the weighted average defuzzification method produced the least APE,
- (3) Models with low numbers of fuzzy sets and rules were capable of low APE,



- (4) Models with their full complement of rules performed better than those whom had had rules taken away,
- (5) Taking an existing model structure and method of model construction, then increasing the number of fuzzy sets decreased the APE,
- (6) The non-algorithmic Experimental Trials (Experimental Trials 1, 2, 3 and 4) produced a lower APE when using the defuzzification method of Smallest of Maximum.
- (7) The algorithmic Experimental Trials (Experimental Trials 6, 7, 8 and 11) showed a lower APE than the non-algorithmic ones, whilst also using a shape based defuzzification method (weighted average method).

#### **A6.4 Experimental Methods: the Other Cost Model Characteristics**

The cost model characteristics are shown in Table 2.1. In Table A6.1 they are linked to the numerical experiments. Having requirements in terms of cost model characteristics can now be linked to the results in Chapter 5. This forms part of the proposed decision making methodology in Section 6.5.7.



**Table A6.1: Cost model Characteristics in the Numerical Experiments and how the  
are Affected (Chapter 4)**

<b>Cost Model Characteristics</b>	<b>Represented in Numerical Experiments</b>
Manufacturing volumes	None
Variety of tasks	None
Repetitiveness of tasks	Inappropriate for rule-based fuzzy logic
Accuracy	Measured by APE, APSD, AAE, AASD
Amount of subjective judgement	Number of fuzzy sets, number of rules, measure of imprecision (Figure 6.3)
Personnel whom operate the system	MATLAB software requires some knowledge of fuzzy logic theory
Detail of input data	The use of crisp numbers used to test the models, fuzzy sets as inputs to the model have not been tested
Estimate application time	Number of variables, number of fuzzy sets, number of rules
Operating costs	Number of variables
System set up costs	Number of variables, number of fuzzy sets, number of rules, number of data points or an equivalent number of decisions making for cost per decision as a potential performance measure

### **A6.5 Cost Model Characteristics for Fuzzy Logic**

Key cost model characteristics, as discussed in 3.8, are introduced in this section for the purposes of describing the advantages of using fuzzy logic for cost model development. The cost model characteristics are summarised under 2 headings: “Personnel involved” and “Data requirements, model development and operating costs”.

### **A6.6 Personnel Involved**

Fuzzy logic has an impact on personnel involved through the following points:

1. In applications involving control, process operators have been asked to supply rules and other information. In this respect fuzzy logic can radically alter the cost model development process by using less resource intensive data collection effort, mainly



through the area of knowledge acquisition, for example interviews. In addition further low cost methods, for example Total Quality Management methods like relationship diagrams, can be used in the process of identifying fuzzy concepts and relationships between them.

2. The whole cost model development process can be completed through fuzzy logic software: MATLAB allows interaction through the command line; customisation of commands by defining defuzzification methods for example; but also graphically through the Graphical User Interface. The GUI promotes an informal trial and error approach as an example of *learning*.
3. Models may be understood on 2 general levels, i.e. high level word description or low level parametric model. The former promotes auditing at tactical or strategic levels and the latter promotes understanding by engineers.
4. The fuzzy logic method can be further simplified by using the form of a checklist of questions asking for key properties of the elements of the Fuzzy Inference System. Phrases found in Table 6.3, in which the meaning of fuzzy sets is explained, can be used in spreadsheet based questions to extract the parameters of fuzzy sets in a user friendly fashion.

#### **A6.7 Data requirements, Model Development and Operating Costs**

Fuzzy logic has an impact on data requirements, for example through the following points.

1. Accuracy of the model developed can be interactive, for example a number of iterations can be easily used to improve the model.



2. Models can be built using expert judgement alone, i.e. no need for historical data (Cox 1994).
3. Models can be built on data points alone and used by experts to extract meaning, with the associated problem of the possible dubious nature of this process (Lo 2003).
4. Fuzzy logic models can be adapted, as in the literature (Jahan-Shahi et al 2001), by changing the fuzzy sets to suit the company, or changes in the description of labour skill levels.
5. Expert driven models can be tuned in using ANFIS architecture, but this requires a TSK type model.
6. Fuzzy logic models are robust. They can use a low number of simple triangular fuzzy sets as inputs and still produce control level accuracy (Yan et al 1999).
7. The number of parameters in a fuzzy set and the number of rules controls computation time for real-time applications. This is not a problem as cost model development is an off-line activity, whereas cost model use is on-line.
8. Rule reduction methods allow for reduced gathering of information by simply reducing the number of decisions required from interviews with experts.

#### **A6.8 Cost model Development Process, Data Identification, Data Collection and**

##### **Data Analysis: Advantages and Disadvantages of Using Fuzzy Logic**

Some of the concerns of using fuzzy logic for cost model development can be summarised through the following observations.

1. Initial gathering of data can come solely from expert judgement.
2. New technologies from other applications can provide a rich source of expertise.



3. Defuzzification methods can produce vastly different output for the same FIS.
4. All fuzzy logic structural elements can be chosen based on subjective judgement.
5. All parameters of a fixed FIS architecture can be chosen based on a “neural network’s” computations.
6. Data analysis proceeds through a series of decisions (e.g. about symmetrical fuzzy sets, structure of rules, modes of fuzzy sets, equally spaced fuzzy sets, centroid or SOM based on smooth or sudden change response).
7. Large number of input variables creates a problem in the size of the rule base.
8. Simplification of complexity through approximating complex data sets using clustering algorithms into a smaller number of rules and fuzzy sets.
9. Fuzzy logic uses its tolerance for imprecision to model.
10. Fuzzy logic through different methods can process imprecision, precision, subjectivity, and data points
11. Fuzzy logic is just as suitable for modelling at the conceptual design stage as at the production stage.
12. Fuzzy logic means that the step of making a logical hypothesis about the general form of the cost estimating relationship is removed unlike other empirical methods.
13. Fuzzy logic can attain good accuracy with simple inputs.
14. Fuzzy logic provides a more meaningful model development process instead of the usual case of several subjective engineering based judgements that are hidden when a final model is produced.
15. Fuzzy logic can introduce possible meaning into a data set via learning methods, i.e. ANFIS and clustering.



16. The ANFIS had bad results against expectations, e.g. increasing the number of fuzzy sets in a model actually increased the error. This was because of the larger number of parameters not being trained properly in the relatively insufficient number of epochs given. For example, an Adaptive-Neuro-Fuzzy Inference System with four input variables and one output variable, with five Gaussian fuzzy sets each, has a total number of parameters of 45.

### A6.9 Experiment Based

A number of tables were produced to facilitate the proposed decision making process. The categories, success and merits of the fuzzy logic method for cost modelling are summarised in Table A6.2.

**Table A6.2: Categorising of Fuzzy Logic Methods**

Method	Structural elements	Success	Cost modelling
Subtractive clustering, TSK formation, followed by linear least squares	Gaussian fuzzy sets, weighted average defuzzification method	Definitive (APE, AAE), large number of data points	Data driven
Adaptive Neuro-Fuzzy Inference System	Gaussian fuzzy sets, weighted average defuzzification method	Definitive (APE, AAE)-Budget (APE), large number of data points	Data driven
Mamdani	Gaussian fuzzy sets, Smallest of Maximum defuzzification method	Absolutely definitive, Order of magnitude APE	Imprecision, expert driven



**Table A6.3: Categories Within the Proposed Decision Making Methodology for  
Cost Engineering with Fuzzy Logic**

Category	Reason for choice of category
Cost model characteristics	This research has structured the cost model development process using data identification, data collection and data analysis methods chosen via the cost model characteristics (Chapter 2).
Type of estimate	Different types of estimates are typically associated with different stages of the product life cycle. Cost estimate types are well known categories used by cost engineers. For example grass roots costs estimates are associated with products after the detailed designed stage.
Application	Application allows the reference to literature based examples in the literature case base
Data or no data	Data or no data makes for fundamental choice of fuzzy logic methods based on expert driven or data driven classification
Universal approximation	Universal approximation provides theoretical information about whether the chosen fuzzy logic structure can approach perfect accuracy.
Rule reduction methods	Rule reduction methods provide essential resource saving options for data collection
Linear / non-linear (Complex / non-complex)	Linear / non-linear provides indication as to the efficacy of fuzzy logic method. <i>This category is arguably dubious as an advantage of fuzzy logic is its ability to operate without initial knowledge of the form of the Cost Estimating Relationship (CER).</i>



**Table A6.3: Continued**

Category	Reasons for choice of category
Literature and numerically based case bases	The use of Equations 4.1 and 4.2, do not provide a complete mathematical proof of which fuzzy logic structures provide the required cost model characteristics. But the universal function approximation methods provide no prescriptive methods to build fuzzy logic models. Therefore the numerical results are added to with an existing case base of literature based examples to help validate any decisions made with the decision making methodology.
Output	The methodology only provides advice, since it is realised a complete proof in performance is not provided by this research. The output provides information on accuracy as regards to APE, and AAE. The advice given is a “good start”.

**A6.10 Observation of Results**

**A6.10.1 Average Percentage Error (APE)**

From these initial observations it is found that the Mamdani method performed least well with a best result of APE of 48.2%; followed by the subtractive clustering based method with an APE of 4.4%; and finally the best method was the ANFIS method with 1.6%. The Mamdani method improved when estimating linear model, Equation 4.2, i.e. a best estimate of 11.9% APE for Equation number 3.

**A6.10.2 Average Absolute Error (AAE)**

From these initial observations it is found that all methods greatly improved in the light of absolute error rather than error as a percentage of the correct result. The figures in Table A5.23 are best considered in the light of the range of the output variable Y (for Equation 4.1 this was, 1020 to 367697.3; for Equation 4.2 this was 29,705 to 2,727,039)



as shown in the columns on the right of Table A5.23. For example the “best model” from Experimental Trials 4 at 56.5% APE was also shown as 15,098 absolute error. 15,098 should be compared to the extremes of the range of the output variable, i.e. 1,020 and 367,697, and also as a percentage of the range, i.e., 4.12% as shown. Similar comparisons can be made for the other models in Table A5.23. The fuzzy logic method, in this light, can be used with some confidence, but with great care at the lower end of the range of the output variable where percentage errors can be high.

It is also apparent from observation of the results that if a “poor” choice is made in forming the method from the fuzzy logic structural elements, using the Taguchi method as a systematic methodology, and hence the fuzzy logic model, then the corresponding results can be very poor. For example cost model 9 in Table A5.7 indicates that the potential estimating accuracy of a poorly selected fuzzy logic model is 1659% APE for the data generated by Equation 4.1. Similarly cost model 10 from Table A5.12 of the ANFIS method is 1644.4 APE for data generated by Equation 4.1; and cost model 1 from Table A5.15 of the subtractive clustering based method is 2103.7% APE for data generated from Equation 4.1.

The best model from Experimental Trials 4 attained a better APE for a linear cost model than a non-linear cost model. Using APE the results for the linear model fell into the “Budget” (or around 20% APE as defined by Section 2.5) category for cost estimates.

- The ANFIS models built by the Taguchi orthogonal array in Experimental Trials 7 showed a range of APE and Average Percentage Standard Deviations. The best model structure performed well at 1.6% APE. The worst model structure was attained at 4041% APE. Jang et al (1997) reported improved results in estimating



given mathematical functions using ANFIS over multi-layer perceptrons showing how a FIS is potentially better than a neural network. The main points are:

- (1) Increasing the number of data points used to train ANFIS improved the accuracy when considering the best model and best structure chosen from the Taguchi orthogonal array.
- (2) Increasing the number of fuzzy sets did not improve accuracy when considering the relative effect of this structural element in a Taguchi analysis (Table A5.13).
- (3) Increasing the number of training epochs did not improve accuracy when considering the relative effect of this structural element in a Taguchi analysis.

Theory shows that the number of parameters to be trained by the network and the number of epochs in training the network interact and are a significant matter. The number of parameters increases with the number of fuzzy sets and hence number of rules.

The clustering produced some of the best results, for example 5% by model 16 in Experimental Trials 8. The clustering models were built systematically again, using a Taguchi orthogonal array. Since it was realised at a later stage, while the models were being built, that there were interactions as shown in Section 5.12, the Taguchi methodology was modified, as described in Section 4.6.11, to test for interactions between variable cluster influences. There were indeed interactions as indicated by the converging lines in Figure A5.14, that contributed to the accuracy.

- It is shown in Table A5.11 that the Fuzzy logic operator, “product” improved results for the Experimental Trials 1-4 best model and best structure, when used for the



“AND” operator and for implication. The results indicated that the product operator gave a general improvement.

Changing imprecision in Experimental Trials 5 surprisingly gave no significant improvements using the APE error measure.

#### **A6.11 Regression versus Fuzzy Logic Models**

The best APE for the MLR was 192.8 (at 150 data points) and the worst APE for the MLR was 437.2 showing a large range of errors, but not as large as the Experimental Trials involving the fuzzy logic methods. It was found that for the non-linear models the Multiple Linear Regression analysis was outperformed by the best structures of the subtractive clustering based method (Experimental Trials 8 and 11) and of the Adaptive-Neuro-Fuzzy Inference System method (Experimental Trials 6 and 7). Examining the plot in Figure 5.7, it is apparent that the method does not improve significantly over the whole range of data points. In contrast for linear models the Multiple Linear Regression was perfect in its approximation.

Figure A6.1 shows the output sets of cost model number 1 in Table 5.15. There are 415 rules in the model (Table A6.4), corresponding to 415 clusters. Because there are so many rules the output polynomials (one per rule) in the Takagi Sugeno Kang model produced by the clustering, are difficult to instantly observe and comprehend. The polynomials appear as a black blur of labels. The labels have been systematically built by MATLAB (e.g. “out1mf118”) and hence have no meanings akin to words in this case. An example is the 118<sup>th</sup> output zero order polynomial that has a constant for each input variable and a constant for the polynomial.



**Table A6.4: Number of Rules per Cost Model in Experimental Trials 9,  
Subtractive Clustering based Method**

<b>Cost Model Number</b>	<b>Number of Rules</b>	<b>Average Percentage Error</b>	<b>Average Percentage Standard Deviation</b>
1	415	2745.1	15635
2	154	6138.2	23864
3	57	38.9	74.9
4	21	71.8	225.4
5	109	71.4	287.0
6	100	16.1	31.4
7	76	47.5	92.4
8	43	27.1	89.7
9	45	35.0	61.7
10	38	44.9	83.6
11	78	7.4	12.9
12	57	8.1	12.8
13	46	53.5	111.0
14	80	35.0	74.3
15	26	11.8	22.4
16	33	4.7	8.1



Figure A6.3: Output Variables in MATLAB

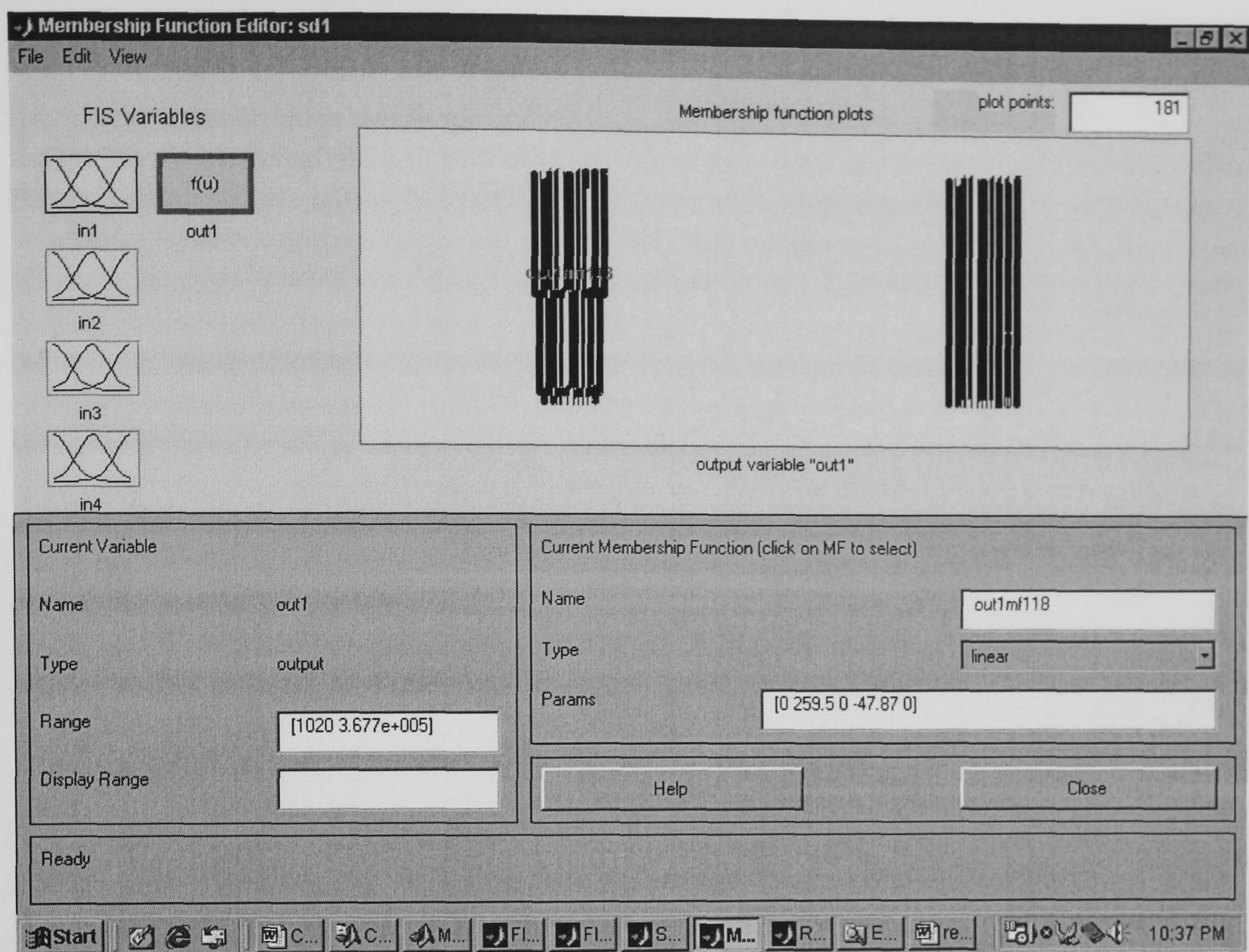
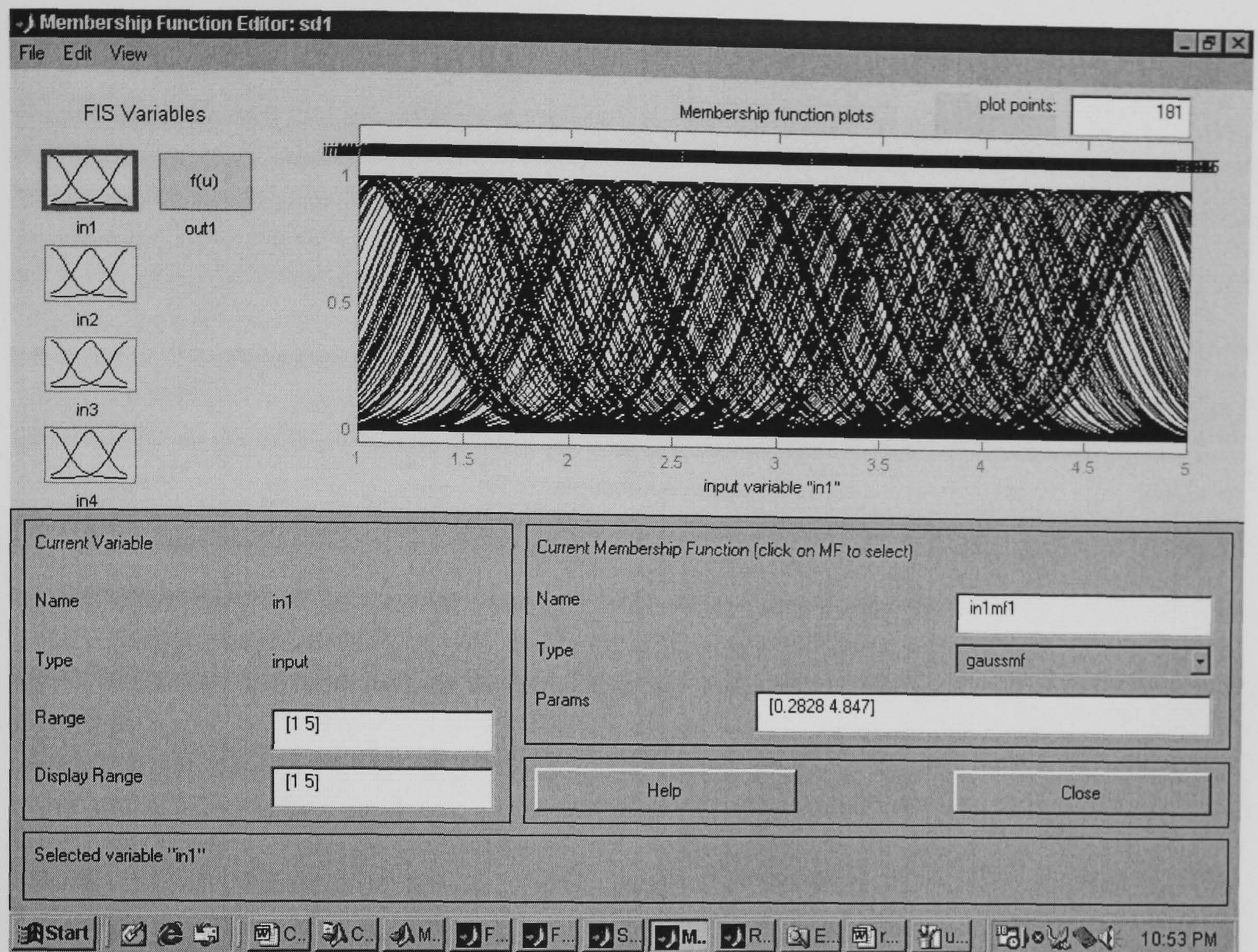


Figure A6.2 shows the input fuzzy sets for input variable X1. There are so many it is difficult to determine whether some fuzzy sets are grouped together in parts of the range or not. It appears there is an even distribution of them.



**Figure A6.4: Input Variables from a Subtractive Clustering Model**



The “best model” in Run 2 of “Experimental Trials 9: Subtractive Clustering” has an APE of 4.7% and 33 rules and clusters within its structure. Figures A6.3 and A6.4 depict the input fuzzy sets and output fuzzy sets of the model. It is therefore apparent that choosing the appropriate fuzzy sets and rules means accuracy is not necessarily better with more rules and fuzzy sets. It is also noticed that the subtractive clustering algorithm differs from Experimental Trials 1-4. Experimental Trials 1-4 increase fuzzy sets and rules systematically and uniformly, whereas choosing different cluster influences leads to choices of different fuzzy logic structural elements in a different sense. This is an important point. Simply increasing rules and fuzzy sets need not necessarily improve matters.



**Figure A6.5: Input Fuzzy Sets for X1 in “Best Model” for “Experimental Trials 9: Subtractive Clustering”.**

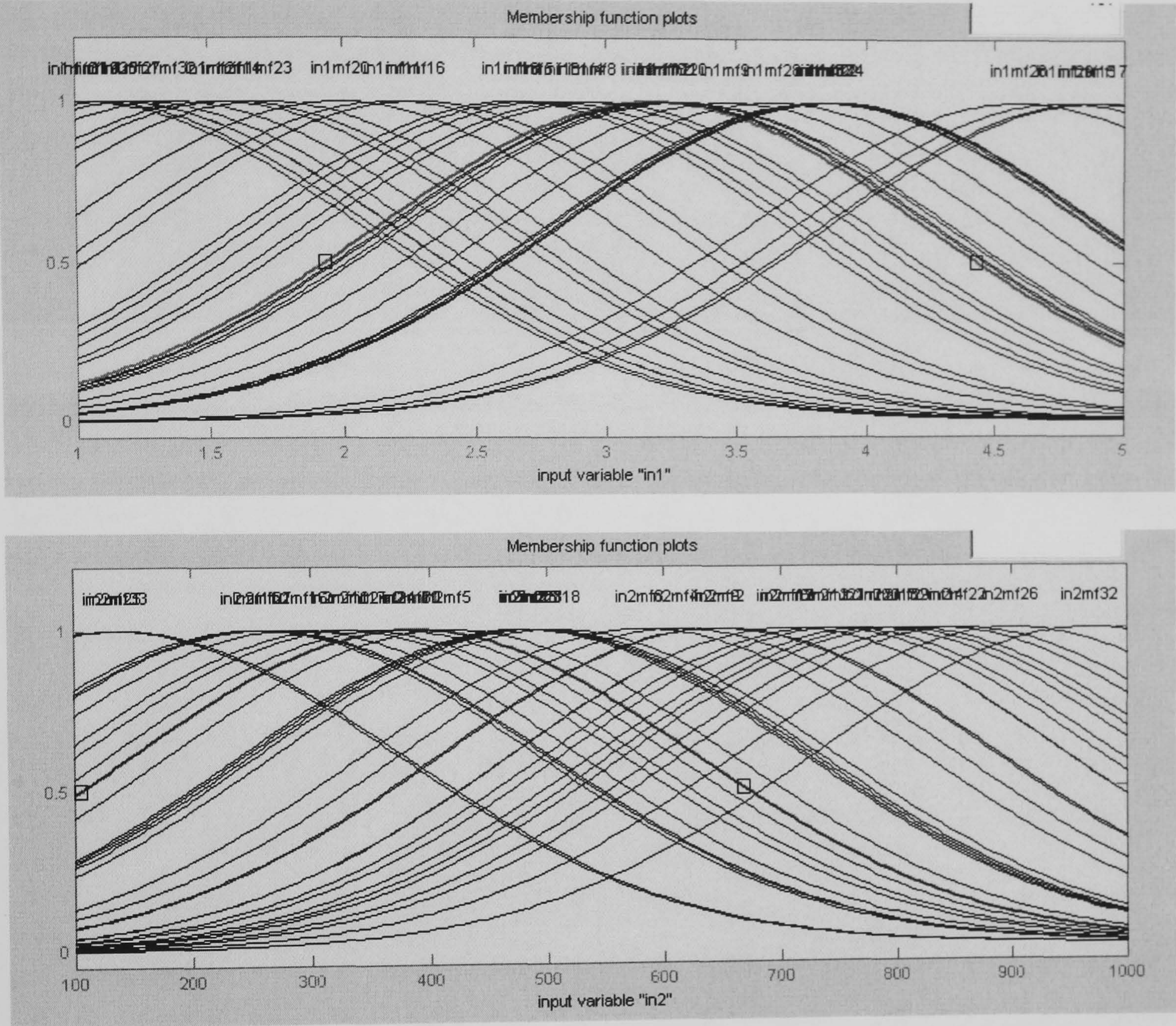
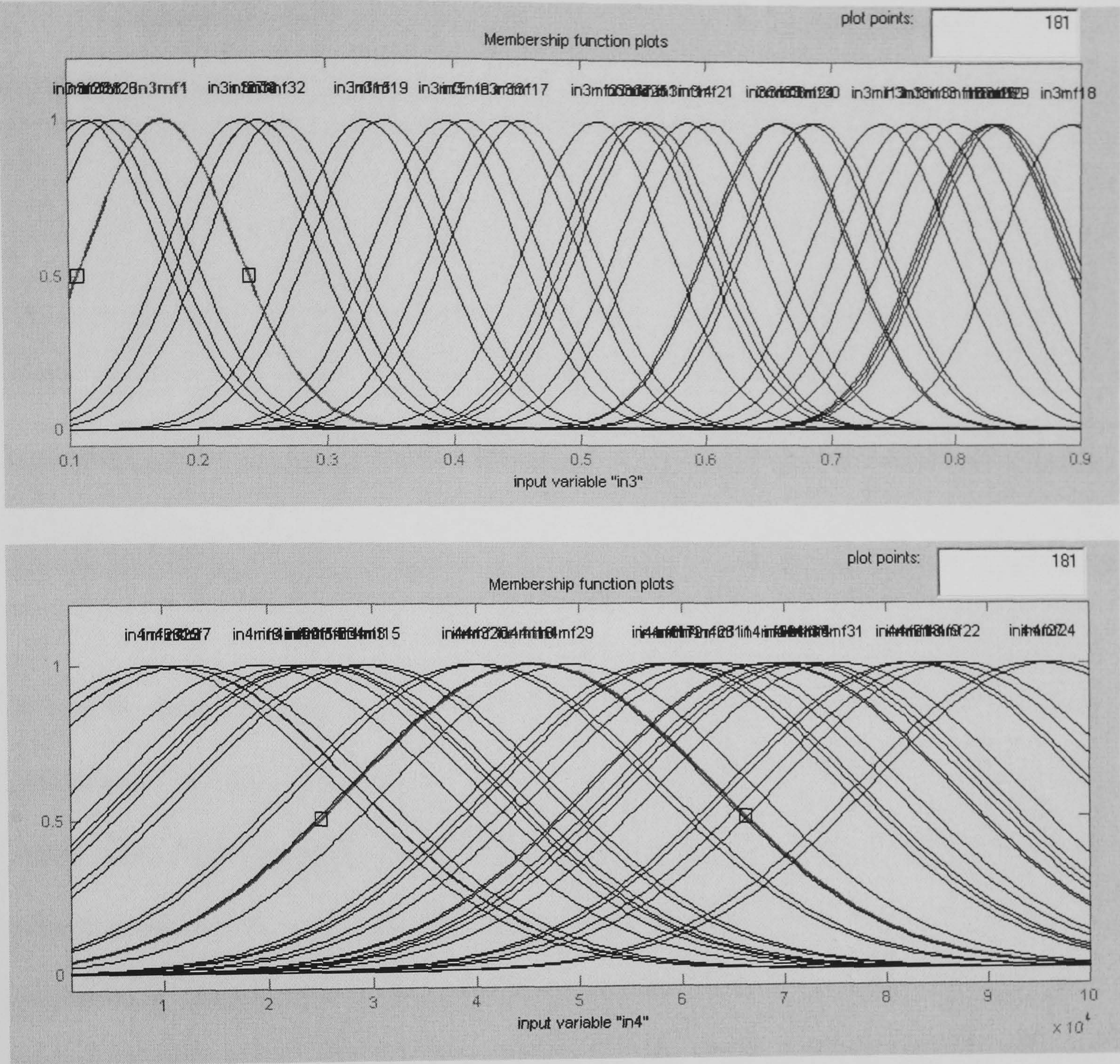


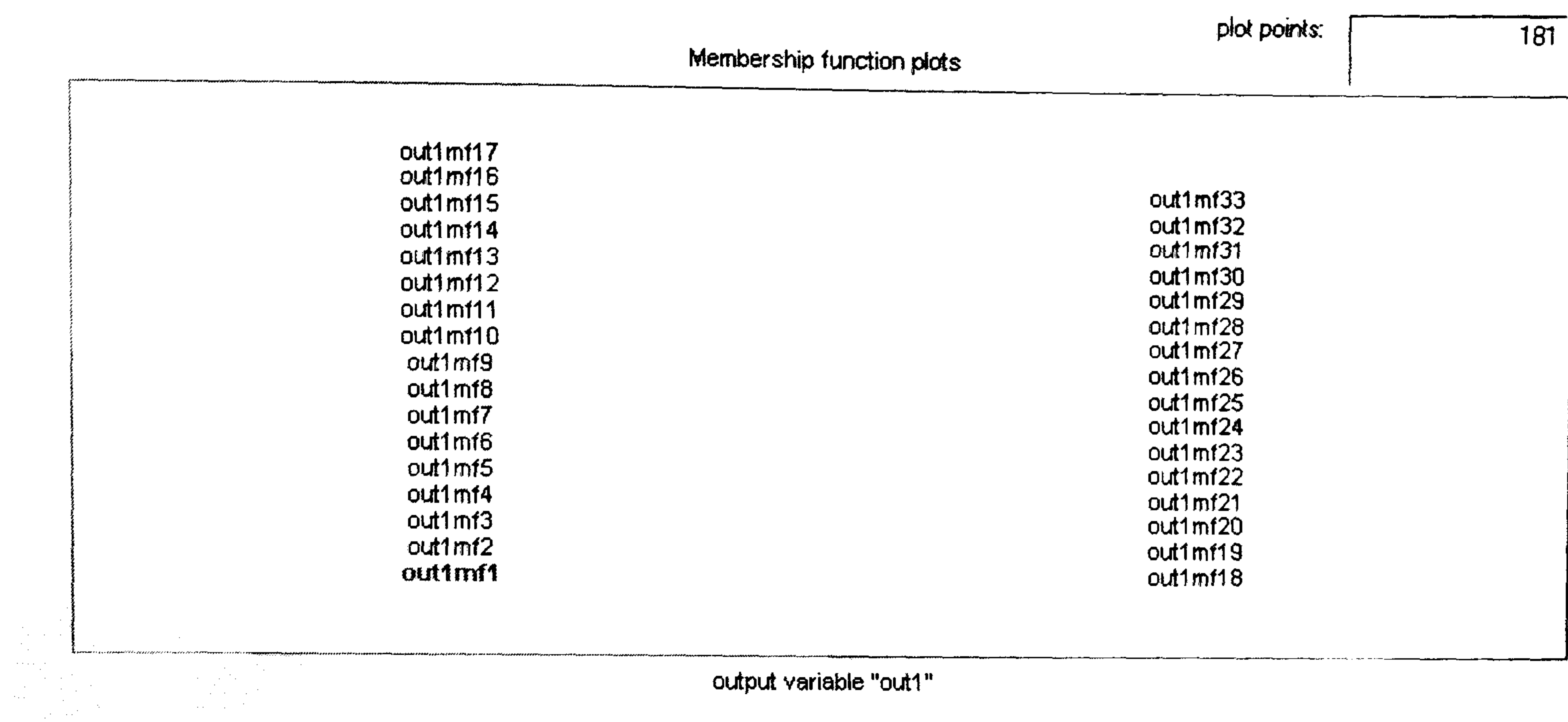


Figure A6.5: Continued.





**Figure A6.6: Output Linguistic Terms (Linear Functions Optimised by Linear Least Squares) from the Subtractive Clustering Based Method.**



It can be seen in Figures A6.3 and A6.4 the effect of choosing different cluster influence. The larger the cluster influence, the more the imprecision of the resulting fuzzy sets. Also the smaller the cluster influence, the less the imprecision of the resulting fuzzy sets. It is important to note that cluster influence can also be chosen subjectively. Cluster influence is described as being rated from a degree 0 to degree 1. The results of changing the degree of influence for a variable can be judged and the influences adjusted accordingly. Cost related parameters can be judged to have such an effect on the other parameters and cost. For example Demirli et al (2003) use a parametric search to identify the best modelling parameters for a subtractive clustering method for the modelling of job sequences.



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